NEWLY DISCOVERED MATHEMATICAL RELATIONS BETWEEN GREEK AND INDIAN ASTRONOMY

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People in India have an experience of Indian astronomy mainly through the Indian calendars, which in the past have been constructed according to various systems, usually that of either the *Ārya Siddhānta* or the *Sūrya Siddhānta.* There are various elements in these calendars, which are called *pañcāṅga* since there are traditionally five elements, the numbers of solar days, lunar days, the *nakṣatras,* the *yogas* and the *karaṇas.* The principle periodicities in these calendars are those of the Sun and Moon. The periods of the planets play a secondary role, of interest mainly in astrological application, while the Sun and Moon are mainly responsible for recording the passage of time. A straightforward comparison with the Western calendar would show first of all that the Indian year begins at a different time, near to the Spring Equinox. This is not just a matter of a conventional difference, for there are deeper astronomical differences between the two systems. One notices for example, that the Indian New Year occurs somewhat later than the true Spring Equinox, and drifts slowly back in time, coming later each year than in the previous year. As I said there are various components in the Indian calendar, and indeed it is really two calendars in one, a purely solar calendar, and a luni-solar calendar which is geared through a system of intercalation to the former. The drift backwards in time of the beginning of the New Year in the solar calendar relative to the Spring Equinox is simply on account of the fact that the 'solar year' is the sidereal year, and is not even intended as an approximation to the tropical year. The Western calendar, either Julian or Gregorian, is based also on astronomical elements, but these elements are very much in the background, so that the equivalence to the tropical year is only apparent as a long term average. The western analogy to the Indian calendar lies rather with the Astronomical Ephemeris, or the Nautical Alamanac as it used to be known, in that both of these are the products of the best astronomy available at the time.

The Indian calendar has been the subject of close study by European scholars, going back to the 1790's. and here one must mention Samuel Davis, Henry Colebrooke and John Bentley, who began the introduction into European scholarship of this subject. Now the interest to the astronomer of this use of the sidereal year is simply this, that the drift in the calendar is a direct record of the phenomenon which astronomers call precession. As a physical phenomenon precession refers to a slow drifting movement of the spin axis of the earth; as a calendrical phenomenon it shows itself as a difference between the sidereal year and the ordinary tropical year. Now since European systems depend largely on the tropical year, and on a system of coordinates which measure the position of the Sun from a point called the equinoctial point where the declination of the Sun is zero, and since European astronomers really never use sidereal coordinates, the early historians felt they could, when faced with the Indian practice argue simply from the displacement in their own time between the Indian and Western calendars, and from the known length of the sidereal year, which they knew from the various treatises such as the *Sūrya Siddhānta* or the *Āryabhaṭīya*, so as to fix the date when the system in any given treatise was instituted. One simply works out the drift and then calculates the year when the New Year in the Indian solar calendar coincides with the true moment of Spring Equinox. Now it is true that one can do this, and one obtains dates usually in the sixth century A.D., or perhaps a little earlier. These dates are not known only to European scholars from the time of Bentley onwards, but are also mentioned by the authors of Indian treatises and the commentaries on them, in connection with the model of precession as fixed in the system in question. They assumed that a quantity of precession called *ayanāṃśa*, negative before this date, and positive after it, was to be added to the Sun's sidereal longitude in order to obtain its tropical longitude.

Now the difficulty in all this, and the pitfall into which historians tend to fall, lies in the assumption that the Indian usage was based directly on observation, *and* that these observations were carried out in terms of tropical coordinates, the same system as used by European astronomers, and that therefore we could infer that the treatises and calendars were based on observations made in the sixth century A.D. or earlier. It simply did not occur to historians that they were dealing with a coordinate system which exists as it were in its own right, and is not simply a curious way of describing observations made like European observations. There were indeed in the 19th century no other examples know to historians based on sidereal coordinates. Ptolemy and Hipparchus quite explicitly rejected the use of sidereal coordinates and European astronomers have generally followed this. There are very good reasons for doing so. But nevertheless it is possible to construct a system of astronomy and a programme of astronomical research in which the coordinates are sidereal. This might be called the astronomy of the night sky, whereas astronomy making use of tropical coordinates is anchored to the movement of the Sun, and to the moment when the declination of the Sun is zero, and this might be called the astronomy of the day-time sky. Of course no astronomy is complete unless these two aspects are brought into articulation, but there are ways of emphasizing one at the expense of the other, and in the Indian systems we have this emphasis on the sidereal coordinates, at the expense of the tropical coordinates. There are chapters in all the Indian astronomical works on the sundial, where it is noted in some treatises that a correction has to be applied to the Sun's longitude in order to obtain theoretical results in agreement with the sundial observation. The *Sūrya Siddhānta* has a full account of the sundial (the type which employs a plane horizontal dial), but not only that work, Āryabhaṭa also gives a similar treatment. Āryabhaṭa however does not mention precession, while the *Sūrya Siddhānta* does so. This reference to precession sometimes seems like a footnote in the treatises, and not as the major link between the day-time astronomy of the sundial, and the night-time astronomy of the solar and planetary periods and *nakṣatras*.

As I said, it did not occur to historians of astronomy in the last century that such an approach was possible. The study of Babylonian astronomy however, beginning with the studies of Epping, Strassmeier and Kugler at the turn of the century, has brought to light a system also based on sidereal coordinates. There is yet a third example in which such coordinates are used, viz. in some forms of Hellenistic astrology. Without going into detail for the moment about Babylonian astronomy, we note that the very fact that it is sidereal helps us to put into a new perspective Indian astronomy. We can see the possibility of consistent astronomy of the night sky, such as we have in the Indian works. Now the point is that it is possible to choose in the night sky various reference stars, giving us a reference point unrelated to the equinoctial point. And if one isn't too fussy about the geometry—indeed geometry is almost absent in Babylonian astronomy—it does not matter at all where one takes the origin. The only elements of "geometry" in the Babylonian systems were the horizon, the meridian and ecliptic, but these must have been regarded only in a very practical way, not really as great circles in the sphere. The zero point—or at least one of the zero points—of longitude seems to have been taken as the point of the ecliptic which culminates simultaneously with the star η Piscium. We shall see in more detail later that the origin point chosen by the Indian astronomers is a sort of spin-off from this long tradition of Babylonian star and planet observation.

Apart from the ephemerides which give the sidereal longitude, we have other—and presumably earlier Babylonian texts, in which the planet's motions are described in terms of the planet's distance from a series of selected stars—33 in number, called by Epping Normal Stars, the first of which was in fact η Piscium. We also have an early (5th century B.C.) text in which the stars alone are described; this famous text, which is on permanent display in the British Museum, gives us simultaneous risings, settings and culminations, and also heliacal risings. In our earliest text the observations seem a bit crude, for there are of course no instruments, and the observer is only told to stand facing south, and to note what stars rise on his left hand, set on his right and face him from the south; a crude but effective means of keeping track of the night sky, and indeed the beginning of precise astronomy.

A really precise method is impossible unless one is a bit more mathematical about it, and for this one has to turn to the Greeks. Hellenistic astronomy is represented in the work of Hipparchus, and one sees there how the stars came to be mapped by making use of this old tradition of observing risings, settings and culminations, but combined with precise notions about the great circles and sphere. This very simple conception was not introduced before Hellenistic mathematics. It was a part of Hipparchus' achievement to develop very considerably the necessary geometry of spherical triangles. With the aid of such purely mathematical results Hipparchus was able to process the observations which he made of risings, settings and culminations, in such a way as to determine star coordinates. Now much of his work is lost. We can examine enough of what remains however to know what a vast effort he made. We have one of his works, and also much quotation by Ptolemy. Hipparchus wrote a commentary on the Phenomena of Aratos, and gives there some 800 distinct numerical data. In the second part of the commentary he gives the precise point of the ecliptic measured from the equinoctial point of his own time (130 B.C.), which appears on the Eastern horizon both at the beginning and at the end of the rising of each constellation. He also gives information about simultaneous risings and culminations, and simultaneous culminations and settings. We must be clear that he does not give coordinates in the modern sense, either Right Ascension and declination, or latitude and longitude. He tells us that a particular star is rising with such and such a point of the ecliptic, or that a particular star is culminating with a particular point of the ecliptic. Note also that the latter is *not* the polar longitude of the star, as used in the Indian texts, for Hipparchus *is* measuring his distances on the ecliptic from the equinoctial point. Hipparchus' information is sufficient, as Vogt showed, to enable one to calculate the full coordinates of 122 stars.

Now we appear to have much more complete information about Hipparchus' efforts in Ptolemy's Star Catalogue. Actually opinions vary according to whether it is supposed that this Catalogue is really Ptolemy's work, or that of Hipparchus. My own view is that it is essentially the work of Hipparchus, one reason being that the detailed techniques needed to fix the coordinates are not described by Ptolemy, who gives only the theoretical geometry of risings and culminations, etc. It is not clear in what way the alignments, which are given by Ptolemy, would contribute toward the coordinates of the Catalogue. I do not think Ptolemy knew those techniques. and that they were lost when the Catalogue was completed, perhaps 300 years earlier, We will return to this Catalogue later because there is something very important I want to explain about it.

Let us now go around to the other side of the picture I am trying to describe to you, and have a look at the last phase of the influence of Babylonian-Indian sidereal astonomy. The last phase is found in certain tables used in Latin astronomy in the 12th to 13th centuries. The continuity with Indian astronomy is fairly direct in that certain Indian texts—one supposes principally the *Brāhmasphuṭasiddhānta*-determined part of Islamic astronomy (if not the *Brāhmasphuṭasiddhānta*, then perhaps some other work, either directly, or through the modifying influence of Sassannian astronomy). One particular set of Islamic Tables of the 9th century, the Tables of al-Khwarezmi, embodied the Indian approach but adapted in various ways to Islamic needs, adapted to the Muslim era, and also recalculated for another longitude than that used in India; these Khwarezmian Tables were used to some extent in Eastern Islam, rather more in Western Islam, in other words Spain and North Africa. Here they were succeeded by another set of Tables, also Indian in spirit, the Toledan Tables. Both Khwarezmian and Toledan Tables were translated into Latin in the 12th and 13th centuries. The Toledan tables, which were more up to date in a sense, had a much greater influence on Latin astronomy, well into the 14th century. It is clear from Latin tracts at the time that astronomers at Paris and elsewhere were trying very hard to cope with this problem of precession and sidereal coordinates. For example, Guillaume de St. Cloud, a leading Parisian astronomer of the late 13th century in Paris, set up an observation in 1290 to determine the precise moment of the Spring Equinox, through an observation of the Sun's declination, and he fixed this time to within about an hour. He then put this date into the Toledan solar Tables, and found that the precession, which he called the movement of the eighth sphere, was 10°13′. Now it is very odd really to try to fix the amount of precession simply by measuring the precise moment of Spring Equinox; but the point is that the Toledan Tables themselves are based on sidereal coordinates, so by substituting the precise moment of equinox and calculating the Sun's true longitude—not the *mean* longitude—you determine the actual drift, like the drift in the Indian calendar. You determine in fact the difference in longitude between the equinoctial point of 1290 and the reference point in the stars on which the Tables were based. In order to find out where *that* is, and to find when the coincidence occurred, we can use modern astronomical parameters to determine the moment of equinox in any year we like, and we can use the Toledan Tables to find the longitude of the Sun at any time. Then by a comparison we can find exactly at what date the Sun's longitude as calculated by the table agrees with that found by modern parameters. Now the Toledan and Khwarezmian Tables, while similar in spirit differ in almost all their parameters, and in particular the four solar parameters are different between the two Tables. It is thus clear that we have a hidden structural feature in this common date, which is A.D. 564. Now, where was the equinoctial point in that year? We find that it is situated some 10′ East of the star ζ Piscium. This star is a famous reference point in Indian astronomy since the time when it was identified by Colebrooke as the star marking the origin of the Indian sidereal ecliptic. In fact in the *Sūrya Siddhānta* we are told that the longitude of that star is—10′, so we have here an exact agreement between the Toledan and Khwarezmian Tables, on the one hand, and the origin of the sidereal ecliptic according to the *Sūrya Siddhānta*, on the other.

Now I must expand a little on this last statement that the longitude of the star is—10′ in the *Sūrya Siddhānta*. What we are given in that work is a sort of abbreviated star list containing 28 stars determined by polar longitude and polar latitude. These stars are described as junction stars of the *nakṣatra*, and *nakṣatra* means one of the 28 small constellations, sometimes misleadingly described as the lunar zodiac, constellations not especially close to either the ecliptic or to the equator; indeed the star α Lyrae is one of the junction stars, and that is quite far north. The choice of junction stars seems to be relatively late in the history of Indian astronomy, compared with the quite ancient references to the *nakṣatras* themselves. These are noted from the earliest times, not only in Indian, but also in Chinese texts. The Indian and Chinese usages clearly have a common origin, as Biot argued. Now in the technical phase of Indian astronomy represented in the *Sūrya Siddhānta* we have not just the 28 constellations but also the regular division of the ecliptic into 27 equal parts of 13°20′ each, and we also have the choice within each *nakṣatra* of a junction star. We have in Babylonian astronomy something similar, viz. the Normal Stars. We can of course speculate that the junction stars occur much earlier in Indian usage than would be immediately justified by the texts. As I said the coordinates of these stars are given in the *Sūrya Siddhānta*, and indeed in quite a few other Indian works of that nature, but not by Āryabhaṭa. Colebrooke, although he knew of differing versions of these coordinates, did make an attempt at identifying them with certain stars, and so likewise did Burgess and Whitney in their edition of the *Sūrya Siddhānta*. There are a number of occasions when it does not appear that any stars correspond exactly to the coordinates, so there are some doubts about the whole strategy of identification. Particularly so when you come to a star described as having zero latitude, and a longitude of either zero or—10′. You look at the star map, in this case the region in the band joining the two fishes of Piscium, where there are only a few faint stars. It is true that there is one star near the ecliptic, viz. ζ Piscium, of 4th magnitude, a star not singled out in Greek sources for any special mention, although it is included in Ptolemy's Catalogue. However this star, if any, corresponds to the Indian coordinates, and this was Colebrooke's choice of identification. His suggestion has never been questioned, but in fact taken for granted by all later writers, both European and Indian.

However we have seen evidence outside the immediate context of Indian astronomy, evidence in the Arabic and Latin continuation of Indian astronomy that the sidereal longitude was taken to be measured from that point near ζ Piscium, and this certainly helps to confirm Colebrooke's broad strategy of interpreting these star coordinates, because we see that the sidereal coordinates of the Sun in the Indian Tables agree with Indian coordinates of the junction stars, and this is an important element of consistency in the evidence and the interpretation so far.

Now I have tried to make you think that there ought to be much doubt about the identification of the junction star as ζ Piscium, and now I want to settle the matter, at least I think I can do so, because I have done some work which helps to explain why this particular point was chosen. When casting about for ideas, I was at the time reading Hipparchus' Commentary on Aratos, a work which marks in a way the initiation of Hellenistic star mapping, and I noticed that he describes the rising of the constellation Aries by the emergence of the star η Piscium (a star which he took to be in that constellation, and which he called the 'forefoot of the Ram'), and he said that the point of the ecliptic having the longitude —11° was on the horizon at that moment. The origin point of the Indian Tables has the longitude-9°20′ measured from Hipparchus' equinoctial point. I examined the star map, and of course easily confirmed Hipparchus' statement and at the same time considered the horizon curve passing through other stars. I had already marked on the map the point diametrically opposite Spica ('anti-Spica')—itself the junction star of the *nakṣatra Citrā*—and I drew the horizon through that for the latitude of 36°, as assumed by Hipparchus for his observatory on the island of Rhodes; this means one is thinking of the setting of Spica simultaneous with the rising of some point of the ecliptic in the East. Then I realised for the first time something about the Indian origin point which I had never seen before, for it was that very point that rose as Spica set. I thought then that if this were true of Spica in the latitude of Hipparchus, perhaps something comparable would be true of other stars at Indian latitudes. I looked at various stars, calculating the longitude of the Hipparchan ecliptic which rises with each star, as a function of geographic latitude. Graphs of this function were plotted for these stars. In fact nothing much of interest happens at Indian latitudes. But something quite unexpected does occur, namely, that four of these curves intersect at one point. These are the curves for α, β Arietis, ζ Piscium and anti-Spica. That means that they lie on a single great circle, a circle which is the horizon for the latitude corresponding to the point of intersection, and that latitude is 36°! Thus you see that the stars α, β Arietis and ζ Piscium rise together with the Indian origin point, while Spica (α Virginis) is setting. This is a quite unexpected but actually plausible result. So the curves show two things, viz. a structural feature of the Greek Star Catalogue, which we did not know about at all, in fact the first piece of internal structure to reveal itself in 2000 years of continuous study of the Catalogue, and also we see a precise connection between this early phase of Hellenistic star mapping and the eventual choice of the zero-longitude point of Indian astronomy. Incidentally, the fact that the alignment relates so clearly to the latitude of Hipparchus is important evidence in the debate about the relative contributions of Hipparchus and Ptolemy.

In Fig. 1 we have a close-up of the star map in the neighbourhood of Aries and Pisces. This shows the Indian divisions of the ecliptic (the *nakṣatras Revatī* and *Aśvinī*), the horizon at the time of Hipparchus, and also the circle of alignment, the Hipparchan horizon at the moment when the Indian origin point is rising.

It is clear that the Greek component, which is undoubtedly present in Indian astronomy of the *Siddhānta* period, must include this subtle feature of the Hipparchan techniques of mapping the stars, and this realization will serve to sharpen questions and hypotheses concerning this most difficult and important problem in the transmission of astronomical knowledge.

The results are given in detail in a long paper, 'Studies in the Medieval Conception of Precession' in *Archives Internationales d'Histoire des Sciences*, **99** (1976), 199-220; **100** (1977), 33-71.

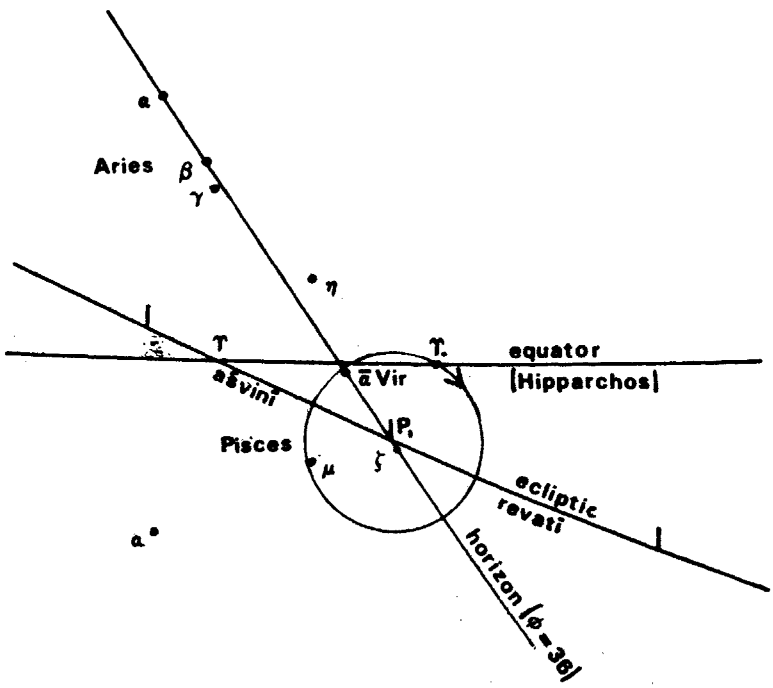

FIG. 1. Showing the stars and coordinate curves in the neighbourhood of the Indian zero of longitude (P_1). The horizon for a latitude of 36° is shown as the circle of alignment of the stars α, β Arietis, α Virginis, ζ Piscium according to their positions in Ptolemy's Catalogue; this circle of alignment cuts the ecliptic at the point P_1, the boundary between the Indian divisions *Revatī* and *Aśvinī*.

(The circle is part of the model of trepidation devised by the Muslim astronomer Thābit ibn Qurra, not discussed here.)