8.g

On the originality of Indian Mathematical Astronomy

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Introduction

Indian astronomy has been the object of intense study by Western scholars since the seventeenth century, before that by generations of Arabic scholars, and of course by Indian scholars themselves over the centuries. Nevertheless we continue to have disputes about the very nature of the subject, illustrating the fact, I suppose, that Indian astronomy is never quite what it seems to be. In the past 35 years, there has been a particularly acrimonious dispute centred on the researches of Roger Billard and David Pingree, both now deceased. I will try to cover what seem to me to be the salient aspects of the matter.

Method of Deviations

Roger Billard in 1971 wrote his *L'Astronomie indienne,* at a time when Pingree's researches were in full spate. Billard's approach was essentially a refinement of what people have always done when approaching ancient or medieval astronomical texts, that is to carry out a comparison with the calculations made by means of modern astronomical parameters, as a 'reality check' in general, and by way of dating in particular. For example Neugebauer & van Hoesen published a collection of horoscopes from Greek literary and epigraphical sources, all of which were dated by means of calculations from modern formulae.[2] Billard's results depended on plotting the 'deviation curves', that is the graph of the ancient mean longitude minus the modern, as a function of time. This was then subjected to a precise statistical analysis, mainly to fix the date of the text. The distinction between 'mean' and 'true' longitude fits the Ptolemaic theory very well, since the 'true' longitude is expressible as a linear function of time (the 'mean') plus a trigonometric expression (the 'equation'), which itself depends on one or more linear functions of time ('anomaly', 'centre', etc.). This distinction between mean and equation is not so clear in modern planetary theory, even when the results of solving the equations of motion are expressed in 'semi-analytical' form, that is a sum of terms involving polynomials and trigonometric expressions. Such a form allows a distinction between mean and equation, but here it is sensible to include in the mean those parts of the trigonometric expression which, although periodic, have very long periods, of the order of centuries. This is particularly applicable to Jupiter and Saturn, which interact in a sort of resonance with a period of the order of 800 years. In the non-Ptolemaic theories of ancient astronomy, such as Babylonian or Chinese, the distinction between mean and equation (or anomaly) has to be established by means of a suitable transformation of the procedure. For example I have converted the summed zigzag model of the Babylonian System B to the form of mean + equation, by representing the zigzag function as a Fourier series.[3]

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[2] Neugebauer (1959).
[3] Mercier 2007b.

Āryabhaṭa

This is a recalculation of the deviation curves such as were published in 1971 by Roger Billard.[4] It is just one of a great many, based on the numerous canons presented in the Sanskrit sources.

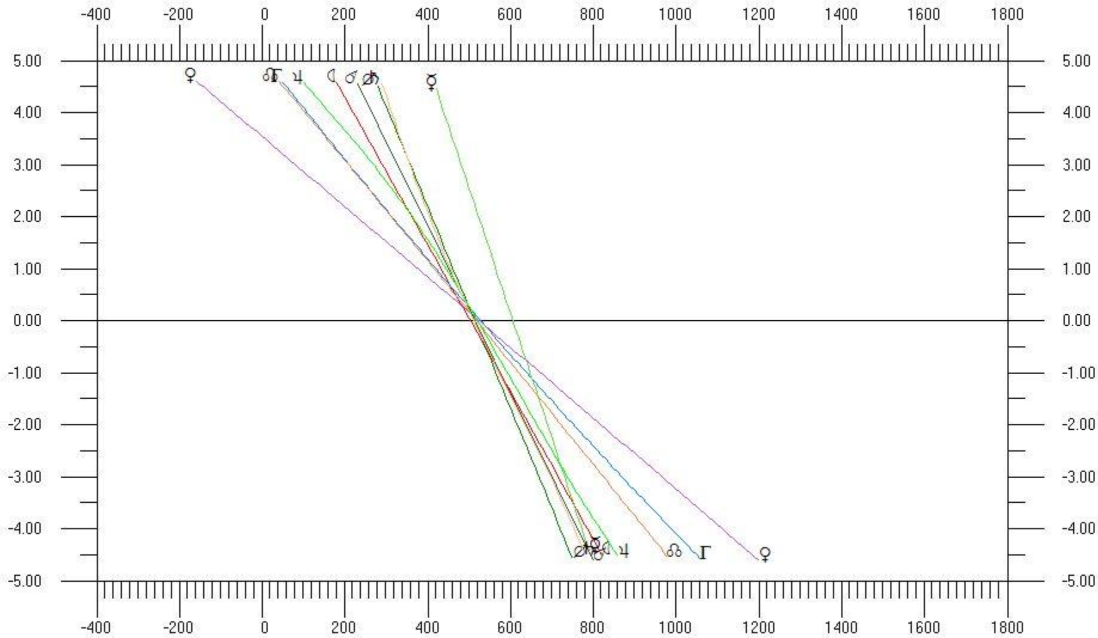

Fig.1. The deviation curves for the canon of Āryabhaṭa

Legend:						
	☉	Sun	☊	Lunar node	♂	Mars
	☾	Moon	☿	**Mercury**	♃	Jupiter
	Γ	Lunar apogee	♀	Venus	♄	Saturn

Each deviation curve represents the excess of the mean longitude calculated according to the text over the modern mean. Plainly the curves form a tight bundle at a year very near to 500; here Mercury must be excepted, and this is generally the case with all canons produced over the centuries, down to and including Copernicus. The general downward slope of the curve is due to the fact that sidereal longitudes are used in the canon, as is the case generally in Indian astronomy. Indeed, the sidereal longitudes are in excess of the (modern) tropical one by the precession, and the downward slopes of the deviation curves is just a measure of that precession, of the order of one degree in 70 years.

The mean longitudes of the canon are fixed by 'revolution numbers' (bhagaṇa), integer valued parameters that determine the number of revolutions over the long base line of 4320000 years. For example for the Moon and Saturn these are respectively 57753336, and 146564. It is simple to observe that if one of these bhagaṇa is changed by ±1, then the deviation curve in 499 (3600 years from the Kaliyuga) will change by $\pm 0.3^{o}$. This results at least on the assumption that the mean longitude is zero at the Kaliyuga, evidently an *a priori* assumption made by Āryabhaṭa. Since the bundle of curves would be noticeably disturbed if any of then were moved by $\pm 0.3^{o}$ we conclude that the values of the bhagaṇa *are the best possible subject to this assumed initial condition*.

The choice of meridian of reference affects the curves, so that for example if computed for a meridian further east, the curve for the Moon would move upward. We may therefore combine the year coordinate in the display shown here with a choice of meridian to find the optimum combination.

Let D_i be the excess of the ancient mean longitude over the modern, the index i running from 1 to 9 (Sun, Moon, lunar apogee, lunar node, Mercury, Venus, Mars, Jupiter, Saturn). The summation may however be restricted to a subset of N items, a subset denoted here by I. Then we may consider either the simple sum of squares of D_i, represented by Q_1, or the sum of the squares relative to the mean of D_i over that selection, represented by Q_0 :

[4] Billard 1971.

$$Q_1 = \sum_I D_i^2 \qquad Q_0 = \sum_I \left(D_i - \left(\sum_I D_i \right) / N \right)^2 .$$

The latter Q is to be used if we want to locate the node relative only to the bundle of curves, in effect disregarding the effect of precession, which determines the general sloping character of the bundle. Billard introduced a simple notation to indicate the subset I, and whether Q is absolute or relative. For example (0 1111 00000) means that only the solar and three lunar deviations are selected, while the first 0 indicates that Q_0 is calculated.

The quantity Q depends on the year t and the meridian ϕ. As a rule there is a 'best' year t_o and a 'best' meridian ϕ_o, when Q will take on a minimum value. Near this minimum point, Q as a function of t and ϕ is represented approximately by the quadratic expression,

$$Q = h_{11}(t - t_0)^2 + 2h_{12}(t - t_0)(\phi - \phi_0) + h_{22}(\phi - \phi_0)^2 + h_{00}.$$

The coefficients h_{ij} are determined by a numerical analysis of the deviation curves, such as those plotted above, but with each of the curves computed for a range of meridians as well as a range of years. For the present example, the canon defined in the *Āryabhaṭīya*, we have in Fig. 2, a plot of the level contours of Q, when the deviations are relative and restricted to the Sun, Moon, lunar apogee and node, the situation denoted by (0 1111 00000). Here the minimum value of Q_0 is located at $t_o = 498 \pm 31.7$, $\phi_o = 80.8 \pm 5.9$; the statistical tolerances are derived from these coefficients. In other words it is clear that the observations upon which the mean longitudes were based were carried out not only in the lifetime of Āryabhaṭa, who was born in CE 476, but for a meridian running through central India.

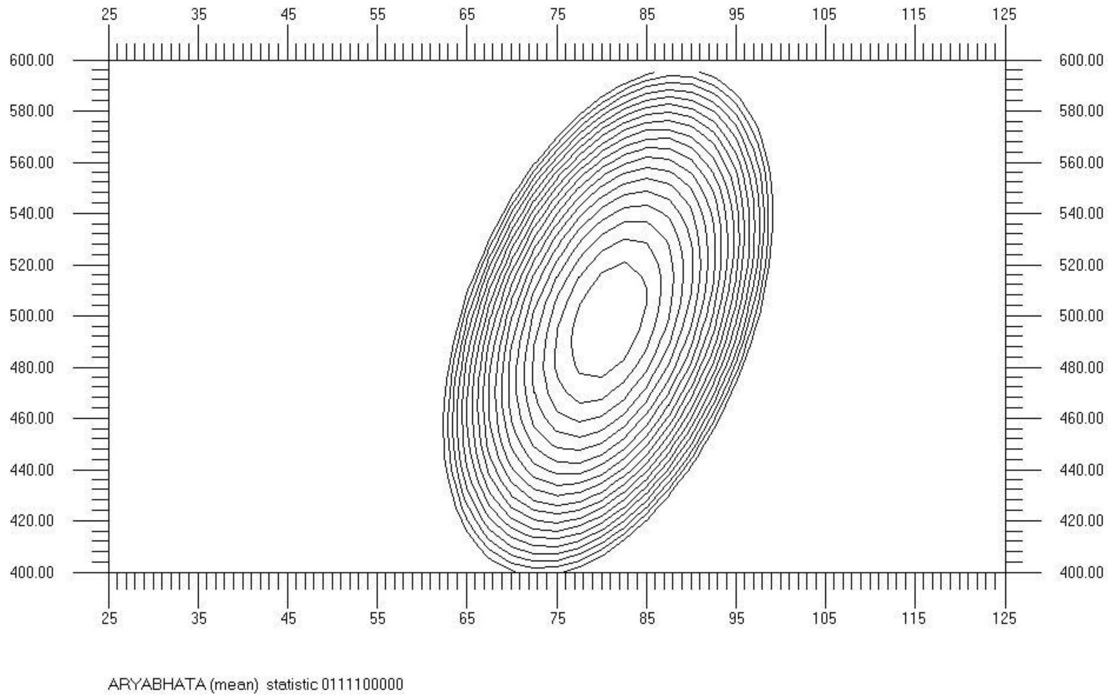

Fig. 2. Āryabhaṭa, level curves of Q_0, (0 1111 00000).

In Billard's survey of the canons he looked only for the optimum year; the extension to the optimum meridian is due to Mercier. Billard always assumed that the optimum year was subject to Gaussian statistics, an assumption which was a serious flaw in the statistical aspect of his work. In my work on the optimum meridian, I realized that we have here a problem of least squares with a small number of parameters (never more than 9), and that there is therefore a considerable departure from

Gaussian statistics. In fact the parameters t_o and ϕ_o are subject to the 'Student' distribution, as it is called.[5]

Pingree, faced with these results of Billard continued to argue that Āryabhaṭa must nevertheless have found a way to derive his mean longitudes from earlier, essentially Greek, results.[6] He was never willing to accept that Āryabhaṭa, or indeed any other Indian astronomer, had been able to make observations, or had been able to reduce these to obtain such accurate mean longitudes. It is however plainly impossible to create mean longitudes many centuries before the year 500 which could somehow be in such very precise agreement with observations at just this time, but not in neighbouring years. For the bundle of curves shows that these mean longitudes were already in disagreement with observations even 20 years earlier or later, a disagreement that only increases, indeed very rapidly, the further one is from the optimum year. That is the meaning of the statistical limit ±31.7. If Āryabhaṭa's mean longitudes were such as to be in close agreement with observations over a very long time base, extending back several centuries, then we might concede that he might have employed a canon that had been created centuries earlier, but that is entirely contrary to what is found. A critical review of Pingree's article of 1976 is given later in this chapter.

At any time in the history of astronomy, from 500 BCE down to the present, astronomical models, or canons, are a mixture of theoretical schemes and empirical conditions. Modern theoretical schemes, depending on relativistic dynamics, are infinitely more sophisticated than anything known in earlier times, and yet even now this theoretical structure has to be fitted to the observed facts, since no theory can yet tell us how heavy the Sun is, or the planets, or where the planets were initially on January 1, CE 2000. So in the time of Āryabhaṭa, although we have a theoretical framework that is essentially Greek in character, employing for example trigonometry that is indisputably Greek in origin (in spite of the important use of the sine in place of Ptolemy's chord), nevertheless, new empirical conditions appropriate to the India of CE 500 have replaced whatever mean longitudes had been used centuries earlier with the same Greek framework. There is, of course, an immensely difficult, and as yet unsolved, historical problem remaining, namely to discover the continuity in the transmission to India of the Greek theoretical framework. It is to be emphasized that the Indian theoretical schemes are different in a number of details from anything known in Greek sources (see Ch. 8.f, above).

DEVIATION CURVES OF TRUE LONGITUDES

In the preceding displays the deviation curves showed the difference between ancient mean and modern mean. It is of course illuminating to compare in the same way the ancient and modern true longitudes. This difference of true longitudes includes the difference of equations, which include terms of short period, such as a day, or a month. Therefore, if plotted over a long stretch of time the difference would just be a blur, so that it requires to be plotted on a rather short time scale.

To be sure, the variations are greater than one finds for Ptolemy's *Almagest*, especially for the Moon, since as one can see, the magnitude of Āryabhaṭa's lunar deviation varies greatly with a period of some 6 months, revealing the need for the improvements that were built into Ptolemy's theory. In Figs. 3, 4 we have the deviation curves for the true longitudes of Āryabhaṭa and Ptolemy.

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[5] Mercier 1987.

[6] Pingree 1976.

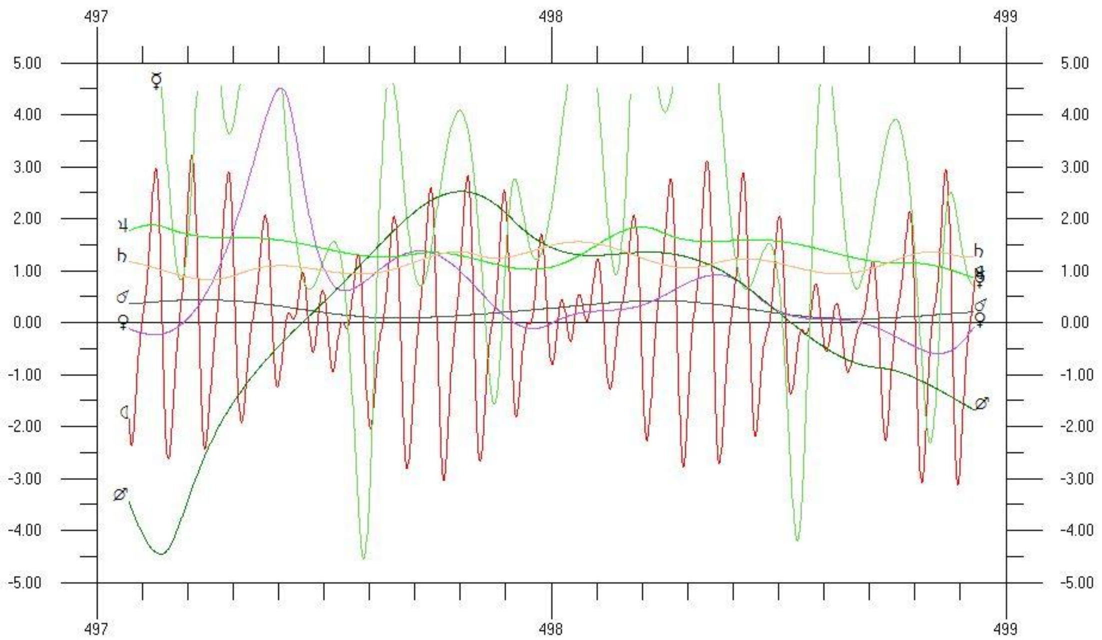

Fig. 3 Deviations between the true longitudes of Āryabhaṭa, and the modern true longitudes, centred on the epoch 499 Mar 21, KY+3600[y].

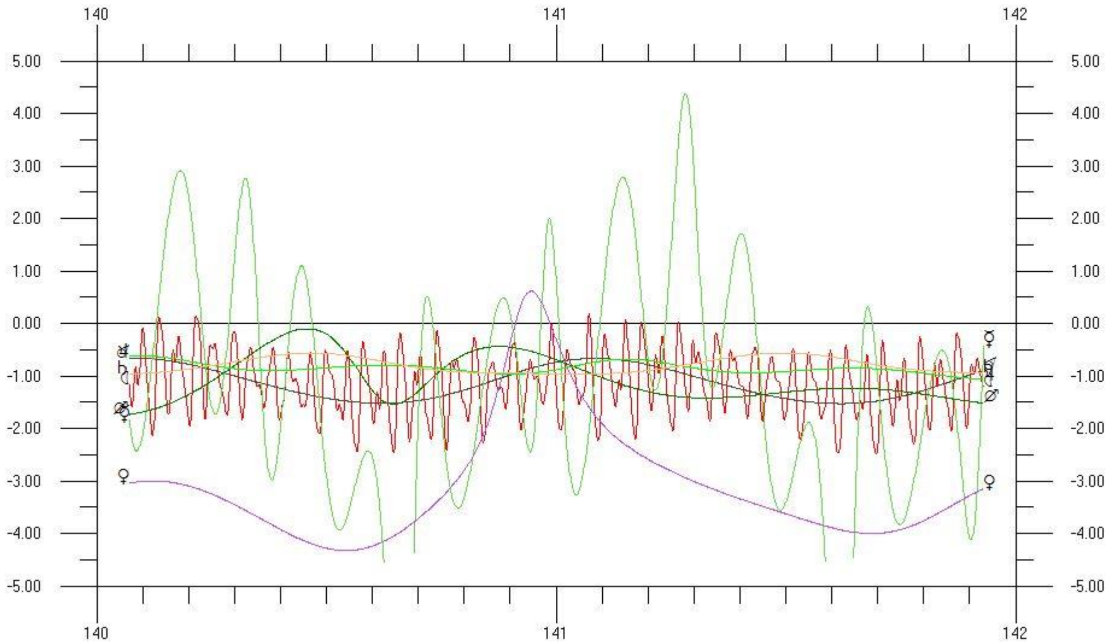

Fig. 4 Deviations between the true longitudes of the *Almagest*, and the modern true longitudes, centred on the year 141.

The general situation of Ptolemy's curves, in his own time, shows that they are too small by about one degree, which is due merely to his dependence on the Hipparchian solar theory. The same display at the time of Hipparchus shows curves averaging close to zero, while at the time of Āryabhaṭa the curves for the *Almagest* are roughly 3 degrees negative. It is noticeable, incidentally, that the variation in the solar deviation curve is smaller with the canon of Āryabhaṭa, in spite of the rather simpler nature of the Indian solar equation.

It has been argued by Pingree that even if the Indians had tried to fix the mean longitudes by observation, that is, somehow by reduction from the observed position, they would not have succeeded, for two reasons. The first is that the equations of the planets were insufficiently precise, and secondly because they had no adequate star catalogue. The latter objection fails because the planets can be logged against the Sun, as in Babylonian astronomy, and to a large extent this was done by Ptolemy as well. As to the first reason one can see from the deviations of the true longitudes that it would be sufficient to log the planet over a few synodic periods in order to arrive at a fair adjustment of the mean in relation to the Sun. Here we have in Fig. 5 the true deviations of Saturn and the Sun. Clearly observations over the interval of years shown here would allow for an adjustment of the mean longitude of Saturn. We should reflect that we do not actually know when Āryabhaṭa wrote the work, only that he chose 499 Mar 21 as the epoch, and that the statistically optimum date is around 510. He may have gone on observing the planets, perhaps up to 530, so as to cover a full range of synodic behaviour of the planet. One should remember also that although we know the date of birth of Āryabhaṭa, and have established when the canon was best suited to observations, we do not know when when it was finally written, perhaps only decades later than the optimum date.

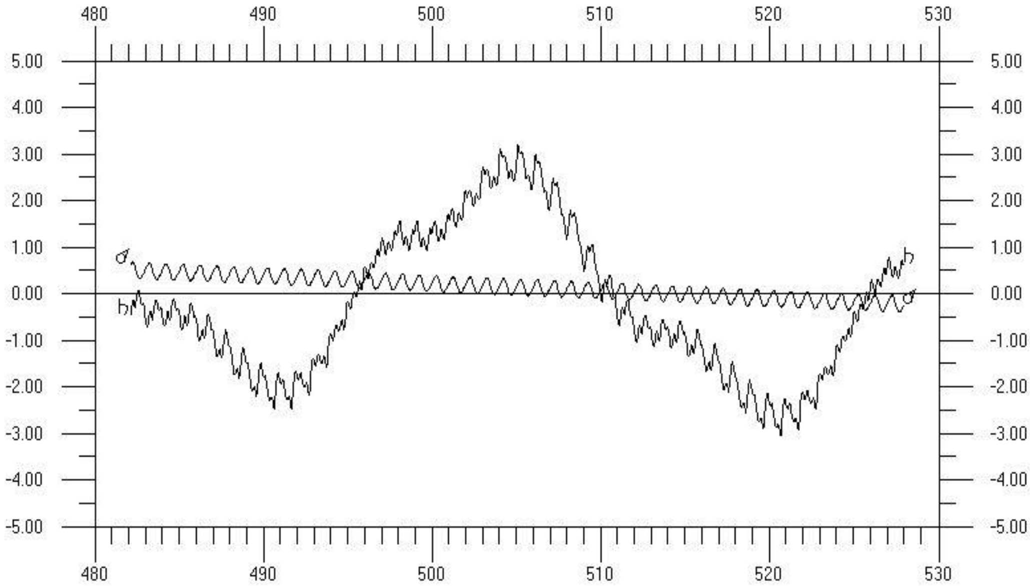

Fig. 5. Deviations of the true longitudes of the Sun and Saturn.

Romaka Siddhānta

Varāhamihirā, an author of the later 6th century, wrote extensively on astrology, and it is in these works that one finds a great many Greek terms in Sanskrit guise, including the Zodiacal signs (*kria*, *tāvuri*, *jituma*, etc.) and other technical terms relating to Greek planetary theory (such as *kendra* < κέντρον). In his *Pañcasiddhāntika* we have accounts of five ('*Pañca-*') canons. This work is not as well organised as it might be, contrasting in that respect with his astrological treatises, and K.V. Sarma has argued that the text as we have it (represented by two extant manuscripts) represents only the first draft of the work.[7] One of these canons, the *Romaka Siddhānta*, provides an illuminating link between Alexandrian origins and the later Indian context.[8] That the work referred to Alexandria is clear enough, for that town is named as Yavanapura, and the interval between that place and Ujjain is given expressly (xv.19):

> Siṃhācārya has declared that the total day count (*dyugaṇa*) commences at sunrise in Laṅkā. The preceptor of the *Yavanas* has said that the day commences for the *Yavanas* 10 *mūhurtas* in the night.

[7] Sarma 1995.

[8] This canon is analysed with varying degrees of success by Thibault & Dvivedi 1889, Neugebauer & Pingree 1970-1, Sastry 1978 (reproduced in Sastry 1989), Billard 1980, van der Waerden 1988a, and Sastry & Sarma 1993.

Since 10 *mūhurtas* = 20 *nāḍīs* = 0;20 days = 8 hours, we have here the equation: sunset+8 hours at Yavanapura = sunrise at Ujjain; that is, sunrise-4 hours at Yavanapura = sunrise at Ujjain. In other words, Ujjain is 4 hours ahead of Yavanapura. While this difference is in poor agreement with the true difference of longitudes (77;50-29;55, or 3;4 hours), it agrees very closely with the entry in Ptolemy's *Geography* VIII.26.13,

> ἡ δὲ ᾿Οζηνὴ τὴν μεγίστην ἡμέραν ἔχει ὡρῶν ιγ δ´ ἔγγιστα, καὶ διέστηκεν Ἀλεξανδρείας πρὸς ἀνατολὰς ὥραις γ ∠ δ´.
> At Ujjain the longest day is approximately 13¼ hours, and is distant to the East from Alexandria by 3 ¾ hours.[9]

He also informs us that Ujjain was the residence of the king Τιαστανοῦ (or Τιβαστανοῦ), Chaṣṭana of the Kṣatrapa dynasty, but at the time of this epoch, it was ruled by the Gupta King Chandragupta I. He also gives the precise longitudes of Alexandria and Ujjain (IV.5.9, VII.1.63, respectively): Alexandria 60;30 Ujjain 117;0, differing by 56;30. Evidently it was not this precise difference, but the round figure 60, that was used in the conversion of the meridian of reference to Ujjain.

If Siṃhācārya then referred the canon to sunrise at Ujjain, this contrasted with Lāṭācārya who, in the preceding verse xv.18, had referred it to sunset at Yavanapura:

> The weekday is obtained from the *dyugaṇa* commencing from a stated point of time, of a particular day at a particular place. Ācārya Lāṭa(deva) has said that the day begins at mid-sunset at Yavanapura.

These two individuals, who are unknown from other sources, were clearly instrumental in adapting the canon from the meridian of Alexandria to that of Ujjain. Now Lāṭācārya is evidently the name of some Master from the Lāṭa country, a region in southern Gujerat, and this suggests the route which the transmission followed, from the port and emporium Barygaza, through the region of Avanti to Ujjain. Under the name Lāṭadeva, he is also noted in I.3 as the commentator on the *Pauliśa* and *Romaka siddhāntas*. This region Lāṭa is fully recorded in Ptolemy's Geography under Λαρικὴ, (VII.1.4, VII.1.62), a region including Βαρύγαζα (bhṛgukaccha, present day Broach, at the mouth of the Narmada), and Ujjain,᾿Οζηνὴ βαςίλειον Τιαστανοῦ, the royal seat of the Western Satrap Cāṣṭana (fl.130).

The epoch of the work is given in I.8 as Śaka 427 complete, at the start of the light half of Caitra, at sunset atYavanapura, starting with Monday. In order to convert this to the Julian date, we might use the calendar implied by the *Āryabhaṭīya*, for example. According to that the mean new Moon occurred a few minutes before sunrise at Ujjain on Monday Mar 21 (J.D. 1905588.75). In the analysis of the deviation curves, to be given presently, this epoch of the *Romaka siddhānta* will be confirmed. The sunset at Yavanapura stated in the verse must have been that of the previous evening, Mar 20. [10] Al-Bīrūnī, in his *India*, quotes from the *Pañcasiddhāntika*, and apparently on the strength of that, states that Lāṭa is the author of the *Sūryasiddhānta*, and that the *Romaka Siddhānta* (of the *Pañcasiddhāntika*) was written by Śrīṣeṇa, but that latter attribution is not made by Varāhamihira, although it is supported by Brahmagupta, when he reviews earlier treatises, notably in Chapter XI (*Tantraparīkṣā*), of his *Brāhmasphuṭasiddhānta*, śl. 48. As a result Dikshit argued that there were two versions of the *Romaka*, one known to Varāhamihira, and another compiled by Śrīṣeṇa some time after Varāhamihira and before Brahmagupta.[11]

The mean motions of the Sun, Moon, anomaly and node are given in *Pañcasiddhāntika* VIII.1-8. There has been some difficulty in establishing the correct mean motions and the correct epoch, but the correct result given here is due to Billard (1980).[12] The following elements are derived from the text. By 'radix' is meant the position at the Epoch date 505 Mar 21, while the 'motion' is the daily rate of motion; both the radix and the motion are in revolutions, not degrees.

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[9] In the earlier edition of Nobbe (1845) has καὶ διέστηκεν Ἀλεξανδρείας πρὸς ἕω ὥραις δ, 'distant to the East from Alexandria by 4 hours', but this was rejected in the new edition of Stückelberger and Grasshof (2006).

[10] This epoch date was given correctly by Sastry 1978 (reproduced in Sastry 1989), correcting Neugebauer & Pingree 1970-1, Part I, p.8, where it is set one day later.

[11] Dikshit 1981:8f.

[12] The mean longitudes presented by Billard in an unpublished article were published by Mercier 1987, and by van der Waerden 1988a.

	radix (sunrise)	motion
Sun	-65/54787	150/54787
Moon	-1984/1040953	38100/1040953
Anomaly	664/3031	110/3031
Node	-56278/163111	-24/163111

Table 1

In the text the radices of the anomaly and node are understood to be given for sunset, but when transferred to sunrise, in keeping with the radices for Sun and Moon, the '609' of the text (VIII.5a) becomes 609+110/2=664, and the '56266' of the text (VIII.8a) becomes 56266+24/2=56278. Expressed in degrees per day the daily motion of the Sun is 360×150/54787, while the year is 54787/150 = 365.24666... = 365¼-1/300, identical to the value used for the tropical year used by Hipparchus and Ptolemy. The Moon's daily motion is 360×38100/1040953, where the large denominator is just 54787x19, so that we rightly suspect that the 19-year cycle is involved. Indeed the implied synodic month is just 19×54787/35250 (= 29.530582 days), which is equal to 19/235 times the year. Moreover 1040953 days = 19×150 years = 2850 years, apparently a 'Great Year' appropriate to this work.

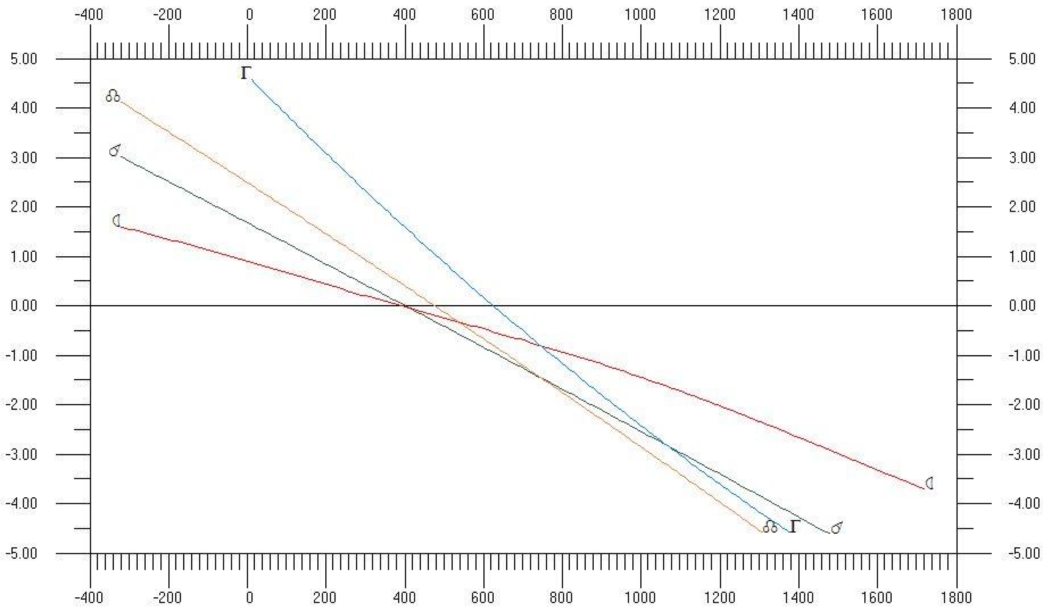

Fig. 6. Deviation curves for the *Romaka siddhānta*

In Fig. 6 the deviation curves for Sun and Moon are computed for an epoch 1905588.75, and the meridian 90 E of Greenwich. Indeed the curves intersect precisely for the year 400.0, and meridian 90.9. Note that statistical limits cannot in this case be established, since of course we have only three parameters for the statistic (111000000), and therefore a *unique* solution in fixing the year and meridian. If we tried a sunset epoch at Alexandria, say 1905588.25, and a trial meridian of 30, the optimum meridian would be - 90. In other words the given parameters work perfectly well for sunrise at Ujjain, and not at all for sunset at Alexandria. The optimum meridian 90.9 is situated some 60 degrees East of Alexandria, by its modern longitude (29;55 E), an interval exactly in agreement with the interval of 4 hours given in the text (XV.19), as noted above.

Sastry (1978) argued that the lunar radix '1984', should be read 10984.[13] He was misled by the application to this canon of the meridian difference between Yavanapura and Ujjain of 7;20 *nāḍi* (equivalent to 44°) which is given in the chapter on the *Pauliśa siddhānta*, III.13.9, but evidently not intended for the *Romaka siddhānta*. However when the deviation curves are inspected after making this proposed change in the mean Moon, the epoch would then have to be for sunset, and for a meridian somewhere *west* of Greenwich. This proposal is therefore excluded. It is considerations of this sort which well illustrate the merits of Billard's deviation curves.

[13] In VIII.4 Sastry proposed changing *kṛtāṣṭanavakaika* (1984) to *kṛtāṣṭanavakhaika* (10984).

Of the various canons that are known to involve the 19-year cycle in one way or another, this canon exhibits the most direct illustration, but is also the least accurate. That is, we may compare it with the *Almagest* on the one hand, and the Jewish calendar on the other. In the Jewish calendar (in the form attributed to the Talmudic sage R. Adda bar Aḥava) the synodic month is the 'classic' Babylonian value, which is extraordinarily precise, while the year is precisely in the proportion 19/235.

The *Pañcasiddhāntikā* gives also a number of details of the equations of Sun and Moon according to the *Romaka Siddhānta*. The apogee of the Sun is given as 75[14]. This, as van der Waerden pointed out, agrees with the pre-Hipparchian solar model of the parapegma of Callipus.[15] The maxima of the equations of Sun and Moon are 2;23,23 and 4;56,48, respectively. To summarise, the Romaka canon appears indeed to be a simple Indian adaptation of an early Greek system.

Brāhmasphuṭasiddhānta

Brahmagupta was the author of two influential canons, the *Khaṇḍakhādyaka* and the *Brāhmasphuṭasiddhānta*. The former depends essentially on the second canon of Āryabhaṭa (that is, the canon presented as the *Sūryasiddhānta* in the *Pañcasiddhāntikā*), whereas the *Brāhmasphuṭasiddhānta*, composed later, is evidently Brahmagupta's own work. The deviation curves for this show that while there is improvement in so far as the various curves agree much better in their slope, which suggests that they may have been to some extent based on a broader time base, yet do not so clearly point to a unique date of composition or observation. Nevertheless they clearly belong to Brahmagupta's period, mid- to late sixth century.

Every Indian canon has a chronological frame, and this is indeed one of the more striking characteristics of Indian astronomy. While the origin of Āryabhaṭa's frame is set at the Kaliyuga, and indeed this legendary prehistoric event is given for the first time a precise sense with his canon, Brahmagupta set the origin very much earlier, so that the base line is 4,320,000,000 years (so-called kalpa), instead of the mahayuga 4,320,000 years of Āryabhaṭa. Brahmagupta's chronological scheme is indicated in outline in the following diagram. here KYaud means Kaliyuga audayika (sunrise), which is the origin of Āryabhaṭa's frame.

Brahmasphuṭasiddhānta zero Kalpa ↓ | KYaud (-3101 Feb 18 Dawn) ↓ | Śaka (78 Mar 16) ↓

days:
← 720,634,442,715 = 0.4567×1,577,916,450,000 → | ← 3179 years →
years:
← 1,972,944,000 = 0.4567×4,320,000,000 →

Fig. 7. The chronology of the Brāhmasphuṭasiddhānta.

The lengthened time base has the consequence, as remarked, of allowing deviation curves that are more nearly parallel, starting from zero longitude at a far earlier date, but also with the much greater revolution numbers a finer adjustment should be possible, compared with the system of Āryabhaṭa. In this case a change of ±1 in any revolution number leads to a change ±0.0003° in the deviation. However it is evident that this was not exploited to the full.

This canon was one of those that came to be transmitted to the Islamic world, where it appears as the Zīj of al-Khwarezmi, early 9th century. Nor does Indian influence end there, for the use of 'Indian' sidereal coordinates continues with the Toledan tables, where the Indian model of precession, ayanāṃśa, becomes 'accession and recession'. Beyond that the Kaliyuga even figures in the chronological basis of the Latin Alfonsine Tables *ca* 1310.

In contrast to the majority of the Indian canons, the deviation curve for the Sun here vanishes around the year 565; in the majority it vanishes around the year 510.

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[14] *Pañcasiddhāntikā* VIII 2.
[15] van der Werden (1988a).

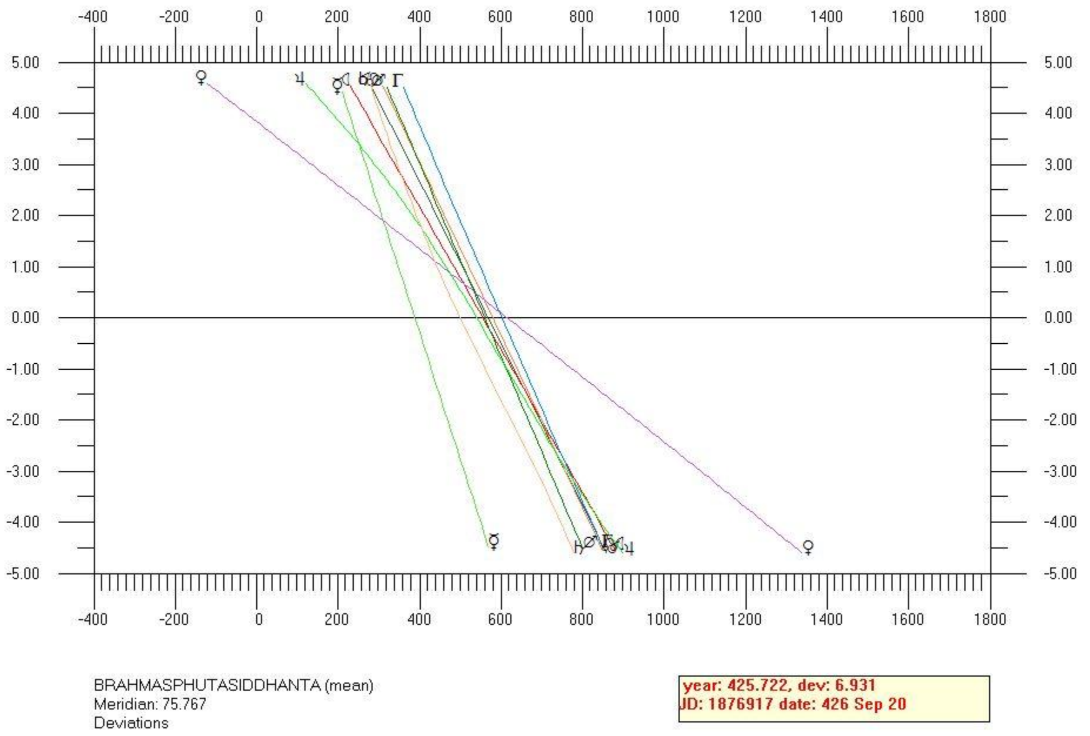

Fig. 8. The deviation curves for the *Brāhmasphuṭasiddhānta*.

Brāhmasphuṭasiddhānta + bīja

The Sanskrit term *bīja*, literally 'seed', refers to a set of adjustments added to the mean longitudes of a given canon. The elements of the *bīja* are linear functions of time, if not simply constant. The result is a new canon whose mean longitudes are still linear functions of time, with new radices and new rates of motion. In the *Brāhmasphuṭasiddhānta* there is such a set of *bīja* in two verses I 59-60. The same set, but expressed in somewhat different terms, is found in a number of other works, such as the *Siddhānta Śiromaṇi* of Bhaskāra II (12th century)[16].

Under the terms of this *bīja*, the mean longitudes of the Sun, Moon and planets are increased by the amount µKY/200, where KY is the number of years from the Kali Yuga, and µ is a multiplier (in minutes) given as follows.

Sun	Moon	apogee	node	Mercury	Venus	Mars	Jupiter	Saturn
-3′	-5′	-2′	-2′	52′	-15′	1′	-5′	4′

For example the mean Sun begins at 0 at the Kaliyuga, but with a rate decreased by 3/(60×200) = 0;0,0,0,54 degrees per year compared with that of the *Brāhmasphuṭasiddhānta*. Thus in the interval from the Kaliyuga to the epoch of Āryabhaṭa, the mean Sun is reduced by what amounts to - 0;54 degrees.

The deviation curves are shown in Fig. 9. The optimum year for the statistic 0111100000 is 1024±46. This is consistent with the date of the *Siddhānta Śiromaṇi* but is evidently wrong for the *Brāhmasphuṭasiddhānta*, a work of the seventh century. Thus it is clear that the two verses I 59-60 are a later interpolation in Book I of that work. This is an excellent example to show the utility of the set of deviation curves. For it clear that the *bīja* was introduced by someone intent are fitting observations in the late tenth –early eleventh centuries.

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[16] *Siddhānta Śiromaṇi, grahagaṇita, madhyagatisādhanādhikāraḥ*, śl 7,8; Śāstri (1929) p. 38. Billard (1971) p. 151, gives references to other works that also give this set of *bīja*.

This is only one of many interesting examples where such a modification was introduced into a canon, with the effect of markedly shifting the date of optimum empirical agreement.

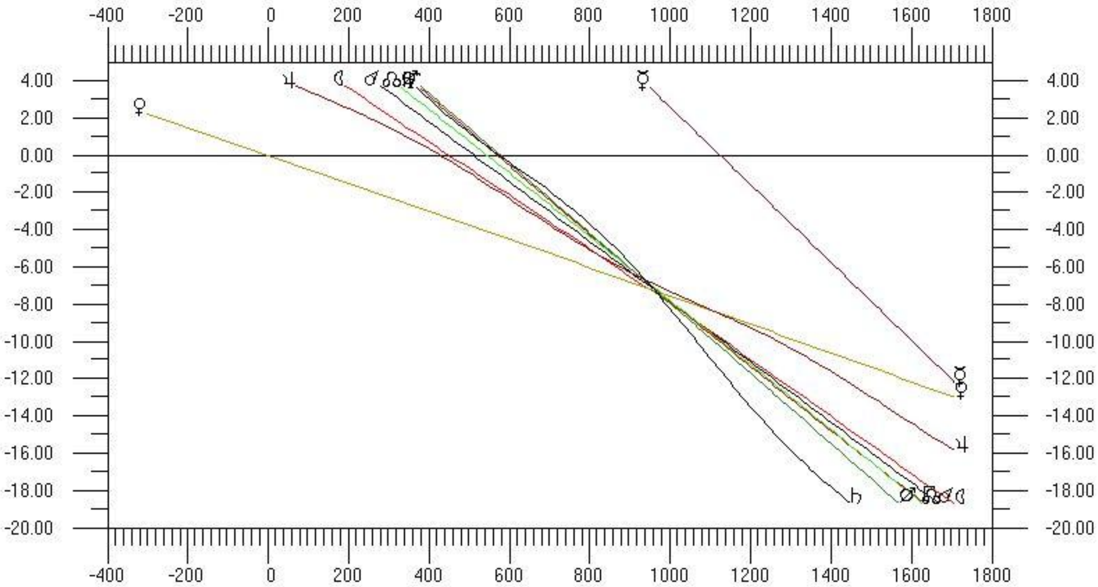

Fig. 9 Deviation curves for the *Brāhmasphuṭasiddhānta+Bīja*

Dṛggaṇita

The canon of Āryabhaṭa is typical of the later canons, insofar as they show real agreement with the observed sky only around one date. We may take the much later *Dṛggaṇita* (‘Observation and calculation) of Parameśvara, for example, whose deviation curves are shown in Fig 9.

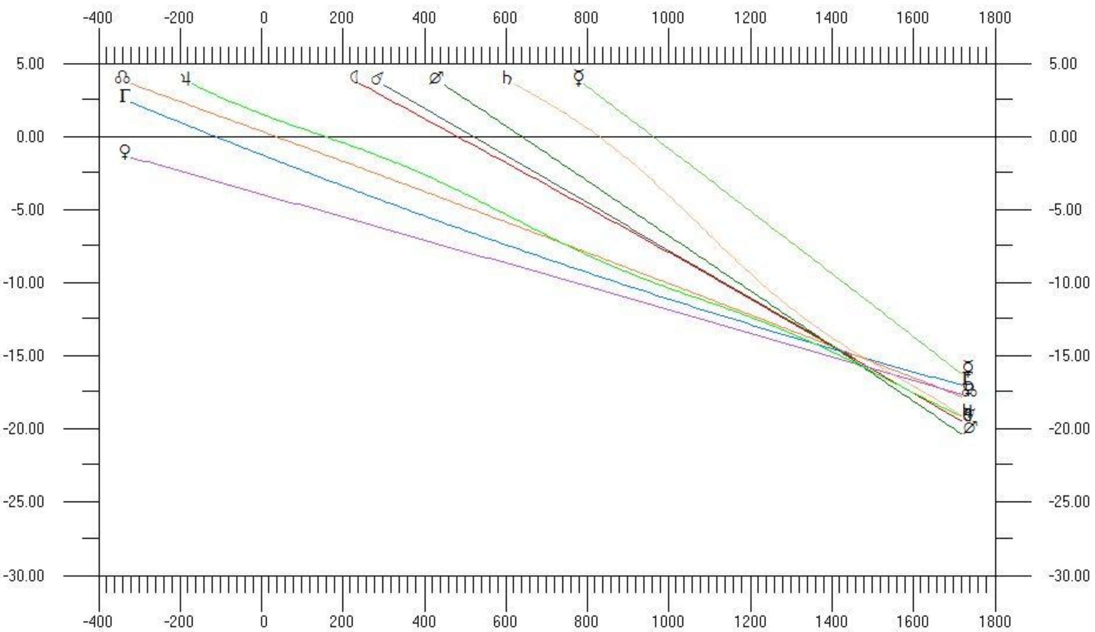

Fig. 9 The deviation curves for the *Dṛggaṇita* of Parameśvara

In spite of the convergence at a later date these deviations do have one essential feature in common with the earlier canons, for the Solar curve passes through zero around the year 510. This illustrates simply that the same sidereal coordinates continue to be used throught out the history of the Indian canons.

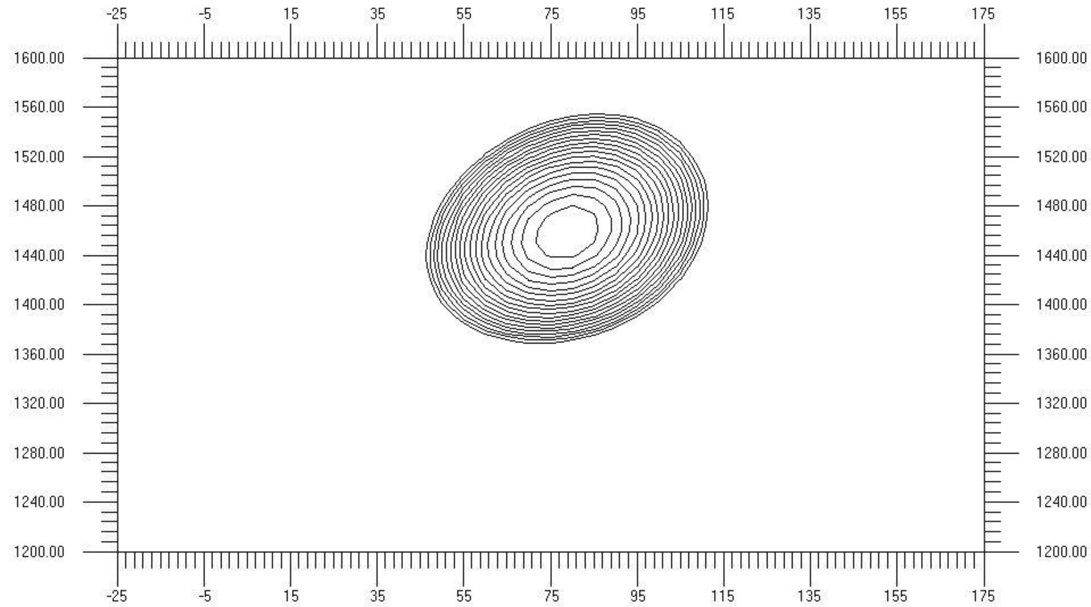

Fig. 10. The level curves of Q_0 for the *Dṛggaṇita* of Parameśvara, with selection (0111101111).

The optimum year and meridian for the stastic (0111101111) are 1458.3±32.0, 78.8±11.1, that is, based on all the deviations save Mercury. The year agrees perfectly well with the known date of this south Indian astronomer, and of course the meridian is that of central India.

Here, however, Plofker sees an opportunity to cast doubt on the merits of Billard’s work. She seems to notice only his remark, that the deviation curves show that the canon is ‘non seulement spéculatif, mais de convergence bien médiocre’. Certainly for an astronomer who claimed to carry out so much observation the final results are indeed disappointing, and we see as is usual with the Indian canons that everything is forced into the yuga system, so that the *bīja* of the canon which define the departure from the canon of Āryabhaṭa depend on time measured from the Kaliyuga. Moreover the node of the curves in his own time ought to be tighter than it is, given the author’s claims. In other words one cannot judge the canon simply by what Parameśvara claims, but only by the end result, which does not live up to his claims. It as if Plofker chose not to examine the results of the analysis of the canon, but to take Parameśvara simply at face value. That is to miss the point entirely of a scientific analysis of the canon.

Dhīvṛddhidatantra

A number of other canons, however, behave more as one might expect from Greek and Arabic examples - that of Lalla, for example.

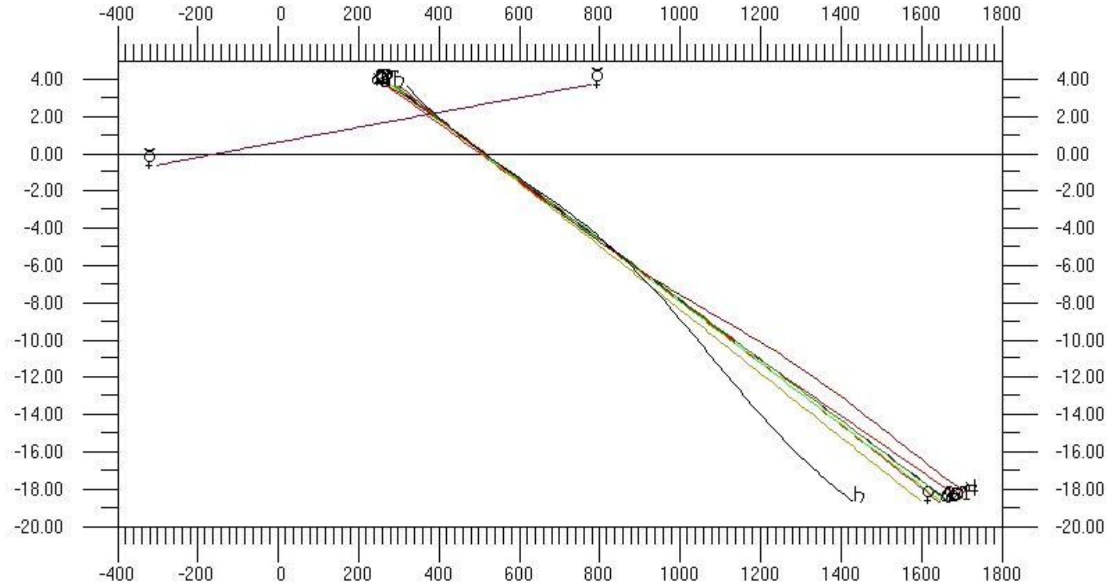

Fig. 11. The deviation curves for the *Dhīvṛddhidatantra* of Lalla

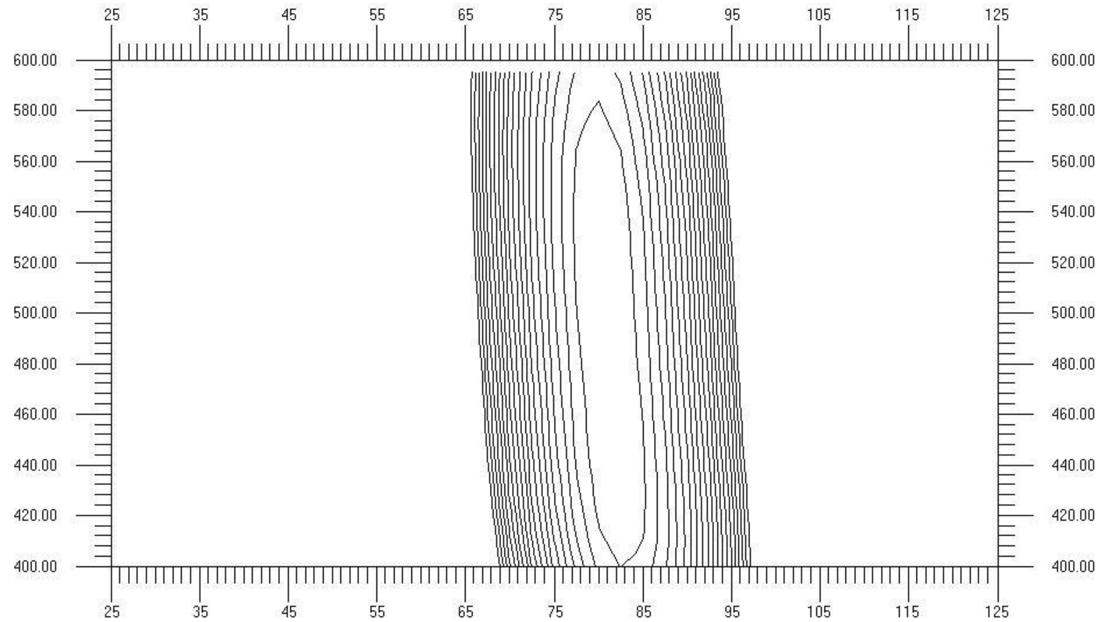

Fig. 12. The level curves of Q for the *Dhīvṛddhidatantra* of Lalla, selection (1111101111)

This canon is defined by a set of *bīja* applied to the *Āryabhaṭīya*, but in contrast to the application noted above made to the *Brāhmasphuṭasiddhānta*, where the increment was propotional to the passage of time from the Kaliyuga, in this case the increment is proportional to the lapse of time from the epoch of the *Āryabhaṭīya*, expressed in form μ(year−498)/250. This destroys the assumption of a common Great Conjunction at the Kaliyuga, in effect doing away with the simple yuga *a priorism* of Āryabhaṭa.

The deviation curves shown in Fig. 11 are remarkable in that they do not now converge to a well defined node. It is relatively unusual - in the Indian context - to find a bundle of deviation curves that run together over a long period. The optimum year and meridian (including all longitudes except Mercury) are 512.2±69.9, 80.7±2.2, where of course the tolerance in the year is quite large Indeed for the Sun and Moon alone (but with the node and apogee), the optimum year is 626±545, in other words the canon would suit any year up to the 11th century. Billard presented an suggestive argument leading to the year 898 as a key year in the construction of the *bīja*, but that was no more than an attempt to squeeeze some information out of the form of the *bīja*.[17] As to the date of composition of this work one is left to speculate, since no date of composition is stated in it. Chatterjee suggests a date between the early 8th and the early 11th centuries.[18]

Plofker, as part of her attempt to discredit Billard's work, made the rather confused remark that Lalla 'mentions *bīja*-corrections referring to the year 748. Billard finds that the parameters ... are optimized for about 898'.[19] However there is no inconsistency here and, in particular, Lalla made no such reference to the year 748; this year arises from speculation based on the denominator 250 in the *bīja* coupled with the base year 498.

Billard made a sharp distinction between these two types of canon, which he called speculative and non-speculative. He saw Āryabhaṭa's approach as governed by a strong *apriorism*, expressed in the set of revolutions numbers, and the assumption of a *great conjunction at a remote Era*, the *Kaliyuga*. Any chance of agreement with observations in earlier centuries was automatically excluded by this *apriorism*.. In order to behave in this way the mean longitudes of the canon were no longer forced all to be zero at a certain remote date. In contrast, the non-speculative canon was constructed so as to agree with observations over a long time base, so that it conformed to our sense of astronomy as natural history.

Now if we examine again the deviations presented by the 'sidereal' Ptolemy, we see something so like the work of Lalla, that we should ask ourselves whether the Indians had received something of the sort from foreign sources. That source could not be the *Almagest* or the *Handy Tables* because the trigonometric principles of the planetary equations were never those of Ptolemy. But we may suppose that Āryabhaṭa had in fact received something like the system presented by Lalla's canon, but which he then 'forced' into a framework of the *yuga* system in order to satisfy the imperative of the *aprioristic yuga*. Although in the existing literature Lalla's system is only available to us from a ninth century text, we should not rule out the possibility that Āryabhaṭa himself had something much like that to start with. After all, the optimum year of Lalla's canon coincides with that of Āryabhaṭa. It makes more sense that way than to assume that someone in the ninth century had been able somehow to resurrect earlier observations so as to obtain a longer time base. So often in Indian astronomy things were not what they seem to be.

There is an interesting comparison to be made with our own sciences of Physics and Astronomy. Physics depends heavily on certain *a priori* constants, like the charge on the electron or Planck's constant. No one would attempt to plot the changes in these quantities over time: the latest measurements just result in a new value of the *constant*, and there is no question of plotting any true alteration in the course of time *in nature*. Our Astronomy on the other hand is regarded more as natural history, so that we are prepared to plot the alteration in fundamental parameters, such as the length of the day, over the course of the centuries. The attitude of the typical Indian astronomer had much in common with our physicist.

Pingree's narrative

By Pingree's narrative I intend to refer to certain key proposals which mark his attempt at an historiography of Indian astronomy.

1. The *Brāhmasphuṭasiddhānta* takes its parameters from the *Pitāmahasiddhānta*, which he assigns to the 5th century.[20]
2. The Persian 'observation' of the solar apogee of about CE 450 was taken from the *Pitāmahasiddhānta*.[21]
3. The parameters of the two canons of Āryabhaṭa were calculated for his time, ca. CE 500 from some Greek tables, and were not based on observations by Āryabhaṭa himself.[22]
4. The date of Āryabhaṭa II.[23]

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[17] Billard 1971: 144.
[18] Chatterjee 1981: XIV.
[19] Plofker 2009: 117.
[20] Pingree 1965, 1970a, 1973, 1978, 1990.
[21] Pingree 1965, 1973a.
[22] Pingree 1976.
[23] Pingree 1970b, 1980.

Throughout these many publications Pingree's writing displays an exuberant self-confidence and panache, which was sufficient to disarm most potential critics. His work took a new turn following the publication of Roger Billard's work in 1971.

BRĀHMASPHUṬASIDDHĀNTA

Pingree has made much of the fact that most of the parameters of the *Brāhmasphuṭasiddhānta* are also listed in the *Paitāmahasiddhānta*, a text which is included in the *Viṣṇudharmottara Puraṇa*.[24] Pingree regards this as the *source*, no less, of the parameters of the *Brāhmasphuṭasiddhānta*, and this assumption forms one of the major fixed points in Pingree's narrative of the origins of Indian astronomy. This *puraṇa*, which belongs to the class of works known as *upapuraṇas*, is taken by Renou and Filliozat, for example, to belong to the period from the seventh to the tenth centuries, and 'probablement dépendant de Brahmagupta'.[25] Pingree, however, would have us believe that the *Paitāmahasiddhānta* belongs to the fifth century, and that Brahmagupta had lifted his parameters from it. However a simple inspection of the deviation curves for the *Brāhmasphuṭasiddhānta* in fig. 7 above shows that the parameters belong to the time of Brahmagupta,.

A further strong point against Pingree's hypothesis is that the set of parameters in the *Pitāmahasiddhānta* is *not* quite complete, because it omits the Era of the work, which is certainly vital information. In the *Brāhmasphuṭasiddhānta* the Era is given as 720634442715 days before *Kaliyuga*. This Era is specified by taking a combination of *manvantaras*, *mahāyugas* and *yugas*, amounting to 0.4567 of the *Kalpa*:

The relevant passage in the *Brāhmasphuṭasiddhānta* I, 26-7 is the following:

kalpaparārddhe manavaḥ ṣaṭ kasya gatāścaturyugatrighanāḥ
trīṇi kṛtādīni kālergo ʾgaikaguṇāḥ 3179 *śakānte ʾbdāḥ* ·26·
navanagaśaśimunikṛtanavayamanaganandendavaḥ 1972947179 *śakanṛpānte*
sārdhamatītamanūnām sandhibhirādyantarāntagataiḥ ·27·

The *Kalpa*, 4320000000 years, is half gone already, and in the second half there are elapsed, after its dawn, 6 *Manu* with *saṃdhi*, 27 *Mahāyugas*, and the three *kṛta*, etc. (*kṛta*, *treta*, *dvapara* : 4x, 3x, 2x *yuga*).

y = 432000 (*yuga*)
g = 4y = 1728000 (saṃdhi, twilight)
Y = 10y = 4320000 (Mahā*yuga*, Catur*yuga*)
M = 71Y = 306720000 (Manu, Patriarchate)

Therefore the number of years elapsed of this second half of the kalpa up to the Kali*yuga*:

$$g+6(M+g)+27Y+4y+3y+2y = 1972944000 \text{ years},$$

to which is added 3179, the number of years from the *Kaliyuga* to the Śaka Era, to obtain the number of years 19729443179 elapsed of the present half of the *Kalpa* up to the Śaka Era. Note that 1972944000 = 0.4567x4320000000.

The solar year is 1577916450000/4320000000 = 365.2584375, therefore we have
365.2584375x1972944000 = 720,634,442,715 days elapsed of the present half of the *Kalpa* to the *Kaliyuga*, which is 588465.75, -3101 Feb 18th, 6 am.

In the *Paitāmahasiddhānta*, however, there is no such passage, so that the Era is unspecified. That means, of course, that *this text as it stands could never have been used for calculations.* Therefore when this point is taken along with the objections centred on the argument about the deviations, it is clear that Pingree's hypothesis about the central role of this work, must fail. While Pingree might have argued that the text of the *puraṇa* has been corruptly transmitted to us, it seems more likely that the copying from Brahmagupta into the *puraṇa* was simply incomplete.

Pingree's view has had wide influence, so that just recently Plofker has repeated uncritically this assumption that the parameters of the Brāhmasphuṭasiddhānta were taken from the

[24] Pingree 1968.
[25] Renou & Filliozat 1947, Tome premier, art. 843.

Paitāmahasiddhānta.[26] Indeed her whole account of early Indian astronomy is an uncritical repetition of Pingree's narrative.

PERSIAN OBSERVATIONS OF THE SOLAR APOGEE

Kennedy and van der Waerden drew attention to Chapter 8 of the *Zīj al-Ḥākimī* of Ibn Yūnis (CE 1003) where, quoting Abū'l-Qāsim Aḥmad b. Mūsa' b. Shākir (CE 850), he reports two Persian observations of the position of solar apogee.[27] The dates are not given expressly, but they are said to be about 165 years apart, the second being earlier than the *Zīj al-Mumtaḥan* by about 200 years. Taking the date of the *zīj* to be about CE 815, the Persian observations would be for CE 450 and 615, apparently; this was Kennedy's interpretation, which, if necessarily inexact, was not unreasonable. Ibn Yūnis reports that the positions of the apogees are respectively 77;55 and 80;0 for those dates. Now it is interesting that these positions of the apogee coincide with those given respectively by the *zīj* of al-Khwārizmī and by the early *Sūryasiddhānta*.[28] Pingree tried to make something of this.[29] For it is well known that the *zīj* of al-Khwārizmī was based on the *Brāhmasphuṭasiddhānta*, although the *zīj* in its extant form has radices computed for the epoch date of the *Hijra* (CE 622). When the apogee is calculated for the *Hijra* from the *Brāhmasphuṭasiddhānta* we find 77;54,32. However since the apogee moves so slowly (480 revolutions in the kalpa of 4320000000 years), the apogee would keep to that value for many centuries, and is essentially the same in the year CE 450 as at the *Hijra*. In any case Pingree saw here support for his notion that the *Paitāmahasiddhānta* could be assigned to the 5th century. Since we now know that the *Brāhmasphuṭasiddhānta* belongs only to the time of *Brahmagupta*, and that the passage of the *Paitāmahasiddhānta* must have been copied from the canon, some other explanation has to be found for the earlier of the two Persian observations. Kennedy and van der Waerden had interpreted the reported observation around CE 450 as indicating activity within Persia, and there is still no reason to reject this. Of course, it may be that the second observation, placing the apogee at 80;0, was due to Persian knowledge of Āryabhaṭa's work in the form of the early *Sūryasiddhānta*. A remark of Severus Sebokht on the Indian numeral otation *in an astronmoical work* that gives us good reason to believe that the work of Āryabhaṭa was known in Persia.[30]

THE USE OF GREEK TABLES BY ĀRYABHAṬA

Here we subject to a detailed analysis arguments presented by Pingree in his article "The recovery of early Greek astronomy from India". There he presented arguments intended to demonstrate that Āryabhaṭa could have derived his mean and true parameters from Greek tables extant in his time, *ca* CE 500.[31] Pingree wrote:

> But if Āryabhaṭa did not observe, how did he arrive at mean motions that produce results more correct for his time than any other? The answer is extremely simple: he used Greek tables of mean motions to compute the mean longitudes for a specific time, and thence derive the rotations in a *mahāyuga*.

This is an article to which Pingree referred on numerous occasions,[32] and which he never revised, although we will see presently that he had good reason to do so. It is essentially an attempt to refute Billard's discovery of the true character of Āryabhaṭa's work, as presented above.

In his discussion he drew on three canons, the *Almagest*, and the two attributed to Āryabhaṭa, that is, the one defined in the *Āryabhaṭīya*, and the one presented by Varāhamihira in the *Pañcasiddhāntikā* under the name *Sūrya Siddhānta*. The mean parameters from which he made his calculation are listed in the following table, which lists all the relevant mean parameters. There is nothing new here, and they are listed only for convenience.

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[26] Plofker, 1009, pp. 67-70, 117.

[27] Kennedy & van der Waerden 1963. The manuscript references are Leiden Or.143 pp.123-5, Paris BN arab. 2495 fol.134; the Paris MS is only a 19th century copy of the MS in Leiden.

[28] For al-Khwārizmī, Suter 1914:9. For the early *Sūryasiddhānta, Pañcasiddhāntikā* IX 7.

[29] Pingree 1965.

[30] Nau 1910.

[31] Pingree 1976.

[32] Notably in Pingree 1978, Pingree 1980.

	Ptolemy(*Almagest*)		*Āryabhaṭīya*		*Sūrya Siddhānta*	
	epoch at 1448638.0	motion in 46656000000 days	epoch at 588465.75	motion in 1577917500 days	epoch at 588465.5	motion in 1577917800 days
Sun	330.75	45985799551	0.0	1555200000	0.0	1555200000
Moon	41;22	614757288630	0.0	20791200960	0.0	20791200960
Apogee	312;33	5197448311	-90	175758840	-90	175758840
Node	317;7	-2471311567	180	-83601360	180	-83601360
Mercury	352;40	190931950101	0.0	64573272000	0.0	6457320000
Venus	41;52	74749631039	0.0	2528059680	0.0	2528059680
Mars	3;32	24450529893	0.0	826856640	0.0	826856640
Jupiter	184;41	3878160391	0.0	131120640	0.0	131119200
Saturn	296;43	1562441331	0.0	52763040	0.0	52763040

Table 2

In his Table 1 Pingree presents mean longitudes according to the *Almagest*, calculated for the *Kaliyuga* (Noon -3101 Feb 17 + 18h, 588465.75), and in his Table 2 according to all three sources for the time of Āryabhaṭa, taken as Noon 499 Mar 21 (1903397). I reproduce here the entries in his Table 1.

Ptolemy: J.D.=588465.75		
	Pingree	recalculated
Sun	314;38	314;38,5
Moon	323;2	323;1,47
Apogee	-	16;0
(Moon-)Node	153;39	163;38,7
Mercury		320;14,44
Venus	--	2;2,48
Mars	301;55	301;54,44
Jupiter	325;4	325;4,13
Saturn	290;48	290;48,13

Table 3 Confirmation of Pingree: Table 1.

Apart from the minor typo in the lunar node his results are confirmed, so that we are clear as to what he mean by the '*Almagest'*.

In his Table 2, he calculates for Noon CE 499 Mar 21 (1903397) the means from the Almagest, and from the two Indian canons of Āryabhaṭa, the *Āryabhaṭīya*, and the *Sūrya Siddhānta* as preserved in the *Pañcasiddhāntikā* of Varāhamihira. The rows are labelled Saturn, Jupiter, etc., for all three canons.

Ptolemy, *Almagest*			*Āryabhaṭīya*	*Sūrya Siddhānta*
Saturn-Sun	48;40	Saturn	49;12	49;12
Jupiter-Sun	188;6	Jupiter	187;12	186
Mars-Sun	7;8	Mars	7;12	7;12
Venus-Sun	356;45	Venus	356;24	356;24
Mercury-Sun	184	Mercury	186;0	180
Moon-Sun+1	283;30	Moon	280;48	280;48
Node-Sun+1	-7;11	Node	-7;48	-7;48
Sun	-2;44,10	Sun	0;0	0;0

Table 4 Corrected version of Pingree: Table 2. The numerical entries are from Pingree's tables, although the labels have been corrected, as explained here.

However in the first column entitled 'Ptolemy' he has in fact listed the quantities shown here in the present Table 4, *now correctly labelled*, that is, the *synodic differences* Saturn-Sun etc. Note that there are also errors of one degree in the lunar means, that is, he has presented the quantities Moon-Sun+1, Node-Sun+1. Since at this moment (1903397) the Sun is zero in the Indian canons it makes no difference whether we call the entries here Saturn or 'Saturn-Sun', etc.

Setting aside the possibilty of deliberate deception, this at least confused and confusing. There is to begin with a distinction, between Pingree's 'Ptolemy' in his Table 1, where he really does intend mean longitudes, and his Table 2, where he lists only synodic differences under Ptolemy: Saturn, etc. In any case, in the spirit of approximation which he asks us to accept, he has demonstrated a rough agreement between Greek and Indian *synodic* differences. This near agreement is however not really very interesting : *for it is in any case inevitable that synodic differences from the two systems system would be in good agreement as long as both are empirically satisfactory*. An agreement between synioodic differences therefore demonstrates nothing about any possible historical link.

Pingree omits from the discussion the mean Sun itself. Here we have a difference of 2;44, which is a significant difference, and which measures the inescapable error between any likely Greek means and the Indian means at this date in 499. For, the synodic differences being in agreement, then this disagreement in the Sun would carry over to the other means. Pingree, it would appear, wants us to ignore this, although it is fundamental to any consideration of whether ot not the Indians copied from a Greek source.

There remain still the differences of the order of 1 degree between some of the Greek and Indian synodic differences. The disagreement of about 2 degrees in the Moon (even after correcting Pingree's arithmetic at this point) just corresponds to the difference 2;44.

Finally, in my extension of Billard's approach, where the optimum meridian is established jointly with the year it is found that for these Indian systems the optimum meridan lies well within India, strongly reinforcing the view that we are dealing with real observational control in India.

Here is his conclusion:

> These tables [Pingree's Tables 1 and 2] do not prove that Āryabhaṭa used Ptolemy, but rather that he used some other Greek source close to Ptolemy. They also demonstrate how trivial Āryabhaṭa's task was once he had a Greek set of tables of planetary mean motions, and explain his apparent accuracy in the early sixth century and increasing inaccuracy as one moves away from his own lifetime; the first reflects the accuracy of the Greek tables, the second the inaccuracy of his assumption of a mean conjunction in -3101.

In fact this 'conclusion' is worthless, for the reasons given above.

In the remaining sections of the paper Pingree draws up a table showing the comparison between the various magnitudes of the equations used in the *Almagest* and in the Indian canons. There is in fact no particularly close correspondence, only a good general agreement as to the order of magnitude, as one would expect since in both systems there is an accommodation to the same observational data. Pingree, however, was never one to let the facts get in the way of a good story.

THE DATE OF 'ĀRYABHAṬA II'

Pingree attempted to refute one of the more unexpected results of Billard's studies, that is, that the canon defined in the work entitled *Mahāsiddhānta* belonged to the early 15th century, not to the mid-tenth, as had been supposed ever since Dikshit's analysis, made at the end of the 19th century.[33]

The work is now attributed to 'Āryabhaṭa II', at least by Dikshit, but this is attribution not supported by the commentators. In my review of the matter I examined quotations from the work by Nṛsiṃha (b. 1586) and Minīśvara (b. 1603) who refer to the work by its name *mahāsiddhānta* or by a reference to the author as *laghvāryabhaṭa*, 'the young Āryabhaṭa'[34]. In that review I also explained how Dikshit arrived at his erroneous conclusion that the work belonged to the late tenth century, and there also refuted the rather glib remarks made by Pingree. He had supposed that the argument was open and shut on the strength of a remark in Bhāskara's commentary, *Vāsanābhāṣya*, on his own *Siddhāntaśiromaṇi* (mid-12th century), where he writes (*Grahagaṇita*, *Spaṣṭādhikāra*, 65)

> *ata evāryabhaṭādibhiḥ sūkṣmatvārthaṃ dṛkāṇodayāḥ paṭhitāḥ*
> for the sake of precision decans were used for the ascensions by Āryabhaṭa and others

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[33] Dikshit 1981:97, Pingree 1970b, Pingree 1980, Pingree 1992, in a reply aimed at van der Waerden 1988b.
[34] Mercier 1993: 10-11.

Pingree, in referring to this, wrote '... Āryabhaṭa II is quoted by name by Bhāskara II in the commentary... ', and indeed if the 'II' were somehow supported by the Sanskrit passage, it would be a an argument that Bhāskara II knew of the *Mahāsiddhānta*. However, it most clearly is not supported, and the reference must be to the earlier Āryabhaṭa.

Anyone who ignored the Sanskrit text, might well be impressed by such an argument : 'quoted by name', indeed ! For the full discussion the reader is referred to my 1993 study.

The deviations are shown in Fig. 13.

It is interesting to consider a near contemporary work by Makaranda (b. 1438), whose canon was formed by a set of *bīja* applied to the later Sūrya Siddhānta.[35] This set consists solely in alterations to the *bhagaṇa* of the later Sūryasiddhānta, and is all the more impressive when compared with the very confused and heterogeneous deviations of that Sūryasiddhānta. For this illustrates once again that Indian astronomers knew well how to adapt a canon to contemporary observations. The deviations are shown in Fig. 14. It is helpful to compare these two results, since no one will dispute the date of Makaranda's canon, while the convergence of the deviations is similar in these two cases.

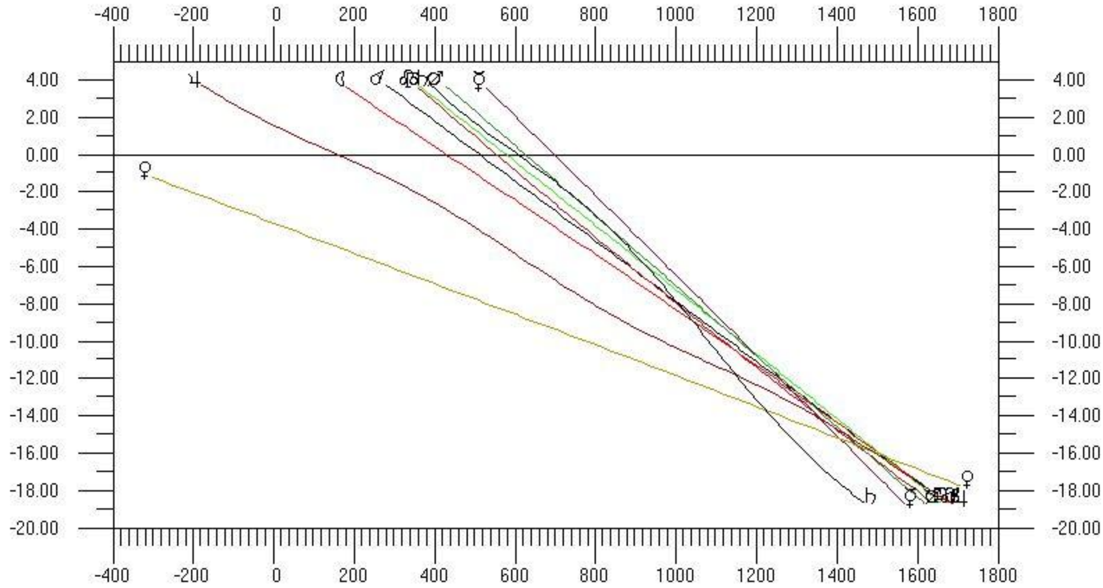

Fig. 13 Deviation curves of Mahāsiddhānta

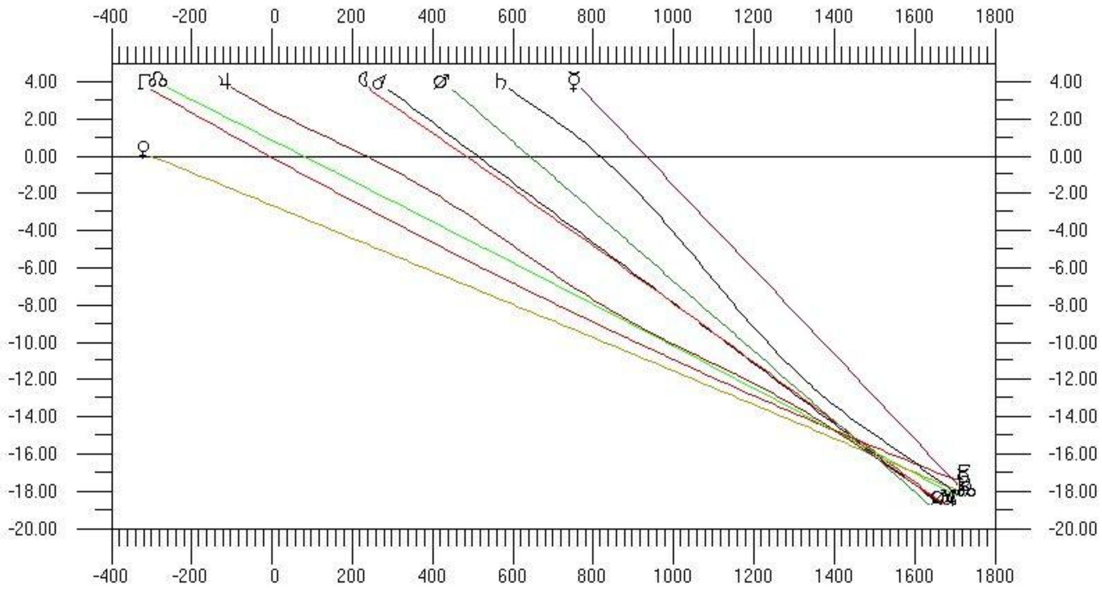

Fig. 14 Deviation curves of Makaranda

[35] Burgess 1860: 22.

It is disappointing to note that Plofker, in a recent volume, has repeated Pingree's error, writing 'we cannot account for the discepancy by hypothesizing that Bhāskara must have been referring instead to Āryabhaṭa I, since he mentions a feature peculiar to the Mahāsiddhānta of Āryabhaṭa II.'[36] Here, like Pingree, *she sees what she wishes to see* in the text, which makes no mention of 'II'. Besides, like Pingree, she ignores the result of the scientific investigation through deviation curves.

DISCUSSION OF PINGREE'S WORK

The starting point of Pingree's reconstruction of the siddhāntic phase of Indian astronomy is his 'discovery' that the mean parameters of the *Brāhmasphuṭasiddhānta* were in fact taken from a text which he dates to the mid-fifth century, the *Paitāmahasiddhānta*, which forms a part of the *Viṣṇudharmottara Puraṇa*. This claim may be dismissed as worthless, for the reasons given above. It is clear that the quotation of these parameters in the puraṇa is a plagiarism. The essentials of this narrative were created by him before the publication of Billard's work in 1971. Billard's scientific analysis of many canons included the proof that the mean longitudes of the *Brāhmasphuṭasiddhānta* were established in the seventh century, and so were certainly the work of Brahmagupta, as he claimed. This destroyed the keystone of Pingree's reconstruction.

Equally worthless is the proposed comparision between the mean longitudes of the *Almagest* and the Indian systems. For it turns out on careful reading of Pingree's articles that not the means but only the synodic differences, have been compared, and so nothing of interest may be concluded. On comparing any two empirically satisfactory systems, whether related to each other nor not, the synodic differences will be found to be the same, inevitably. The difficulty with Pingree's account is that *according to the words* of his account he has compared the means, not the synodic differences. I can only conclude that he allowed himself to be confused, which I am afraid is not untypical of his approach to much of the history of astronomy.

Pingree has always been adamant that the Indian astronomers never seriously carried out observations, and in this he has simply followed the consensus, which goes back to Colebrooke. He never attempted to meet head on either Billard's argument, or my extension of it to the meridian determination. Indeed he simply ignores that level of scientific investigation. It is difficult - to say the least - to reconcile his attitude with the evidence presented by, for example, the *Dṛggaṇita* (Figs. 9, 10), in which all the mean longitudes have been so well adjusted to a certain date in the 15th century, and for which one can demonstrate an optimum meridian passing through central India. However we have to ask ourselves whether Pingree ever understood the force of such demonstrations. He might also have considered the fact that the Indian calendrical systems have maintained a good correspondence with the true state of the heavens, as one may judge from the numerous dated inscriptions with their records of solar and lunar eclipses. In fact, the remarks in the foregoing sections show that his determined attempts to evade Billard's conclusions only ended in failures of both scholarship and calculation.

Sidereal Coordinates

If we are to begin to follow the transmission of astronomy from the Middle East to India we have to consider the use of sidereal longitudes in these two distinct contexts, for the Indian canons (almost) invariably use that system. While Ptolemaic astronomy uses tropical longitudes throughout, we know that frequently elsewhere in the Greek context sidereal coordinates were used. Ptolemy, who was obviously aware of this convention, gave his reasons for rejecting it.[37] Indeed, a great many Greek horoscopes are known which used sidereal coordinates, a fact which seems clear enough even though we are so often ignorant of the precise steps that were taken in the calculation.[38] For in the long interval between Babylonian methods, and the establishment of Ptolemaic astronomy, there were various systems and canons of which we have only the roughest understanding (see Ch. 6.a here). However, after the *Almagest* was established, it is clear that horoscopes and other results based on it continued to

[36] Plofker 1009, p. 118, n. 89.
[37] Almagest III.1 (Heiberg I 193.11).
[38] Neugebauer & van Hoesen 1959, Jones 1999.

use sidereal longitudes, evidently by way of continuing a well established habit. Ptolemy's longitudes were converted to sidereal equivalents by adding a motion of the solstices according to a zigzag model of precession preserved for us by Theon of Alexandria. This presumably satisfied the need felt by astrologers, in spite of Ptolemy's rejection of sidereal longitudes.

There are examples from this period for which Neugebauer argued for the use of sidereal coordinates, when in fact the results do not really seem to compel this interpretation, but in spite of that it is broadly true that sidereal longitudes were the popular and preferred style. For example, I have some doubts about whether the Demotic Stobart Tables and the evidently related Demotic Papyrus Berlin 8279 (see Ch. 3.a, *sub* notes 204 and 210), use sidereal coordinates, in spite of Neugebauer's insistence.[39]

THEON: MOTION OF THE SOLSTICES

The model recorded by Theon as having been used in horoscopes, is reported to us in Theon's *Short Commentary on the Handy Tables of Ptolemy*. He records the rule:

$$\text{Sidereal} - \text{Tropical} = 8 - (y+128)/80$$

where y is the year in the Era of Augustus.

This is (apparently) intended to be applied to the longitudes found from the *Handy Tables*.[40] Theon in fact specified a zigzag rule for this precession, with limits of ±8 degrees, the effect of which is shown in Figs. 15.[41] A number of 'five-day' almanacs were calculated from the *Handy Tables* with this correction., for example that in P. Heid. inv. 34, for the years CE 348-350, which presents Saturn, Jupiter, Venus, Mercury starting with the year 672 (Era of Philip). The sidereal longitudes, which are 1;40 in excess of those computed by the *Handy Tables*, may be accounted for by Theonian precession.[42] Jones notes a number of others among the Oxyrhynchus papyri, with dates earlier than this, in the range 14 BCE to CE 307. When the material is sufficiently well preserved to allow for a computational check they all agree with the *Handy Tables* together with Theon's conversion to sidereal longitudes.[43]

If the Theonian precession is added to the Ptolemaic tropical longitudes we obtain the deviation curves shown in Fig 15.

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[39] Neugebauer 1942, Neugebauer & Parker 1960-9, discussion in Vol. III, Part 1, pp. 232-5, edition in Vol. III, Part 2, Plates 75-8; Neugebauer 1975:567, 456, 785-8.

[40] Tihon 1978, Ch. 12, text 236, translation 319.

[41] This zigzag behaviour is sometimes referred to as 'trepidation', but that term is used too often in a loose journalistic spirit, instead of being confined to the Alfonsine context where it properly belongs. In later Indian and Islamic contexts the term 'accession and recession' is close to the Sanskrit, Arabic or Latin terminology. Copernicus also referred to accession and recession. Theon only referred to a movement of the solstices, postulated by the old astrologers.

[42] Neugebauer (1956), Burckhardt (1958).

[43] Jones (1999), pp.45-6, 215-227.

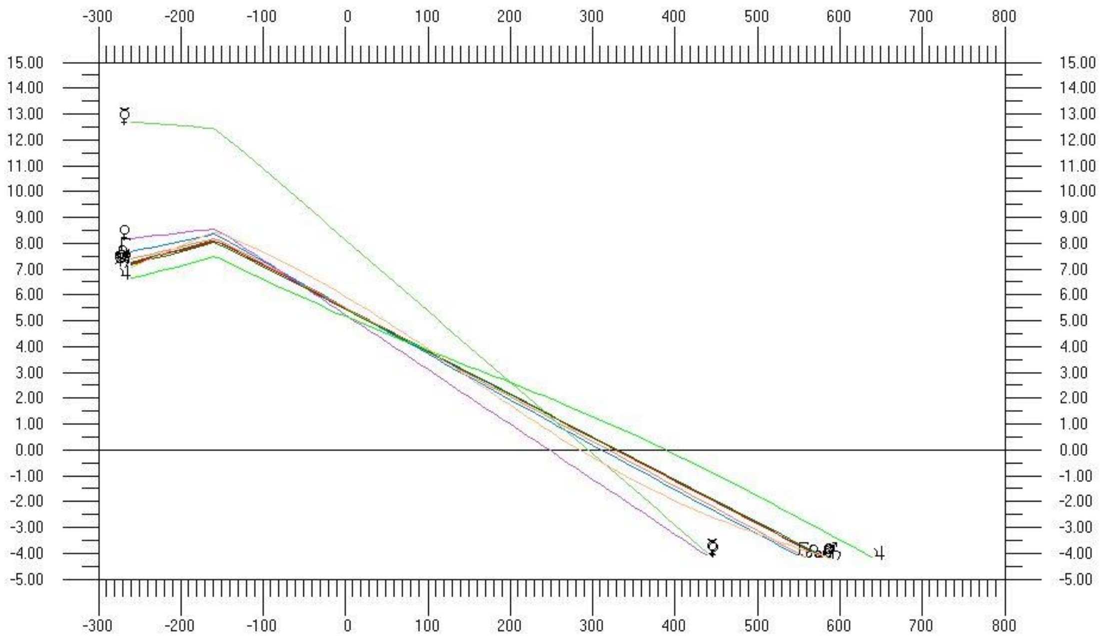

Fig 15. Deviation curves for Ptolemy + precession (Theon)

The curves in Fig. 15 should be compared with those of Lalla's canon, Fig. 11. The two bundles have essentially the same slope, but with a difference of nearly 3 degrees. By way of comparing the slope, note that the daily motion of the sidereal Sun found in this way is virtually the same as that used in the Sanskrit sources. The daily motion of the Ptolemaic Sun is 360/(365;14,48), while the daily motion of precession is 1/(80×365.25); the difference is the sidereal daily motion of the Ptolemaic Sun.[44] This difference corresponds to the sidereal year 365;15,33.66 days,

$$\frac{360}{365;14,48}-\frac{1}{80\times 365.25}=\frac{360}{365;15,33.66},$$

whereas the sidereal year used by Āryabhaṭa or Lalla is 1577917500/4320000 = 365;15,31,15. The two rates of motion differ by only some 2 secs p.a.

The missing element in Theon's account is any information as to which star, or point in the starry sphere, marked the origin of sidereal longitudes; some such point is essential to the system. In fact nothing was expressly stated by Theon on this matter, nor by Ptolemy when he mentions the sidereal year.[45] In an earlier article on precession I suggested that the point opposite to Spica (α-Vir) may have been that origin.[46] One can see from the above bundle of deviation curves that that of the Sunis zero around the year CE 328. Taking star coordinates from Ptolemy's *Almagest*, where we have the ecliptic longitude of α-Vir as 176;40, or 174;0 in the time of Hipparchus (-127), then by the precessional correction of Theon's model, one degree in 80 years, the position of α-Vir in CE 328 would be 179;41. We may suppose with some confidence that α-Vir was the reference star, but calculated according to the Hipparchian star list, and with the rate of precession that of Theon. By modern computation the tropical position of α-Vir is 174 in the year -149, and 180 in the year 285, but while such a calculations are interesting, the historical argument depends on the positions assigned to the stars by Hipparchus.

In the discussion below of the Indian sidereal ecliptic it is shown that for an observer at latitude 36° the star α-Vir sets simultaneously with the rise of the equinoctial point in the time of Hipparchus; this is evident in Fig. 22.

Strangely enough, there is now an official Indian adoption of this very point as the origin of sidereal longitudes, in spite of the clear use in the medieval canons for the point near ζ-Psc. I have discussed this modern tendency elsewhere.[47] It happened that a number of Indian scholars, notably Venkatesh

[44] It is in fact arguable whether or not Theon's precession, 1 degree per 80 years, should be referred to the Alexandrian year of 365.25 days or the Egyptian year of 365 days, but the resulting numerical differences seem unimportant.

[45] Almagest III.1 (Heiberg I 193.11).

[46] Mercier 1976-7, Part II, p.52.

[47] Mercier 2007a.

Bapuji Ketkar (b. 1854), were of this view, and following them, N.C. Lahiri persuaded the *Calendar Reform Committee* in 1954 to adopt this officially, and it is now to be found used in every Indian Pañcāṅga (popular almanach). In these popular calendars the tropical coordinates are now take from modern computation, but converted to sidereal longitudes by means of modern precession theory, but with anti-Spica as the sidereal origin. Ketkar looked for support from the *Pañcasiddhāntikā*, and proposed an emendation to the usual reading of xiv.37, so that the star would be at the midpoint of the 14th *nakṣatra*, placing it in effect at 180. Apart from that, we note that in the later *Sūrya Siddhānta* this star is again given the coordinate 180. Generally, however, in the Sanskrit texts, the star is placed at longitude 183.

THE STANDARD SCHEME OF THE MOON

The Pap.Ryl.27 of the Rylands Library, Manchester, contains a procedure for the computation of the true longitude of the Moon, and its node. This text has been much discussed, and one may consult the recent publication of the Oxyrhynchus Papyri both for a summary of the contents, and useful tables facilitating the computation.[48] This model has been given the name Standard Scheme by Jones. It is a simple zigzag model, working like Babylonian System B, and of course the lunar longitude is sidereal. The importance of the scheme lies in the fact that we have here the explicit procedure for finding the position of the Moon, and not just the result for a particular date, such as we can obtain from the many horoscopes. In the course of Neugebauer's analysis of the numerous horoscopes he noted the difference between the longitude given in the horoscope and that by modern calculation for the date, but this yields a wide scattering of differences and this is discussed below.

In order to situate the Standard Scheme in the general context, one that permits an easy comparison with the trigonometric models, I derived for it an equivalent mean longitude and anomaly, and a mean node. For the sake of argument the meridian of reference is assumed to be that of Babylon. From these we can plot the deviation curves, which are shown in Fig. 16, along with the curve for the sidereal Sun, and precession, separately. Unfortunately, we lack at this time the corresponding scheme for the Sun. It is at once clear that the deviation curve for the Moon fits nicely with that of that Ptolemaic sidereal Sun, and that the curve for the Node does so as well. However the curve for the apogee shows that there is a difference of some 9 degrees. Now the important point about the Moon is that in this scheme the lunar longitude is sidereal, and yet coincides almost exactly with the sidereal Moon obtained from the *Handy Tables*. Since a sidereal longitude is measured from a conventional and arbitrary origin in the star sphere, we have here the striking result *that the sidereal origin assumed for the Standard Scheme is the same as that used by the Theon's model of precession.*

[48] Jones 1999, where references to earlier work will be found.

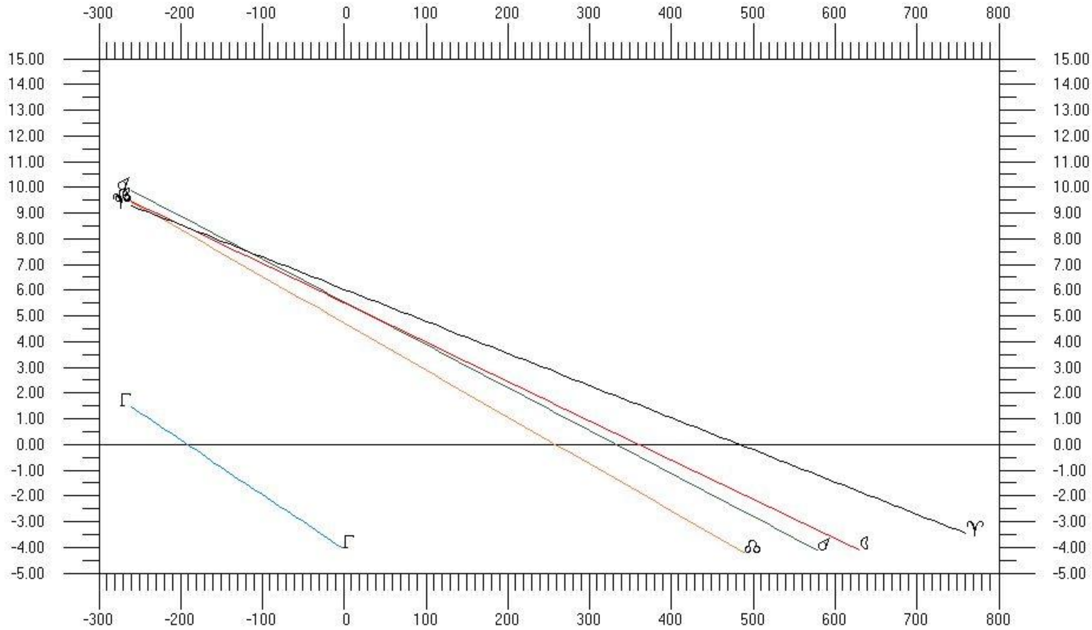

Fig 16. Deviation curves for the Standard Scheme of the Moon, together with that of the Sun from Ptolemy + precession, and precession separately.

GREEK HOROSCOPES

Neugebauer & van Hoesen published a collection of horoscopes from Greek literary and epigraphical sources, so providing a wealth of material bearing on Greek methods of calculation.[49] Those of the first and second centuries plainly give sidereal longitudes, and Neugebauer regularly determined the differences from modern calculations. Such differences are a feature of the earlier pre-Ptolemaic horoscopes. However the collection also included many horoscopes from the fifth century, and these are demonstrably based on Ptolemy (probably the *Handy Tables*), although you would never know this from their account. Neugebauer, although devoted to calculation in general and the study of Ptolemy in particular, failed to notice that these fifth century horoscopes were Ptolemaic in origin. This fact, although it has been known also to Anne Tihon and Alexander Jones, has never actually been published.[50]

In Fig. 14 the individual points represent the excesses over the modern calculation, for the horoscopes of the first and second centuries. They include many horoscopes provided by Vettius Valens, whose *floruit*, if not known exactly, must have been in the latter half of the second century CE.

[49] Neugebauer & van Hoesen 1959.

[50] However Neugebauer 1975:293 n.8, does allow that the astrologers of the fifth century used Ptolemy's tables.

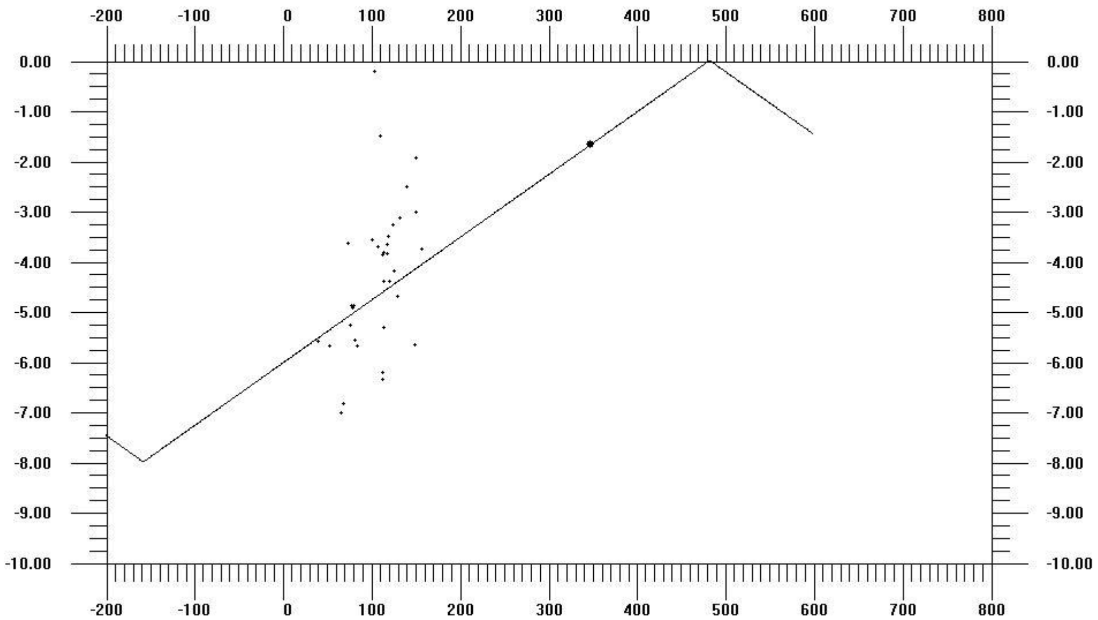

Fig. 17. Precession by Theon, with sidereal excesses from horoscopes, including the almanach of CE 348/9 (P. Heid. inv. 34).

The points in Fig. 17 are computed from the various horoscopes published by Neugebauer. The horoscopes are listed in Tables III A and III B. A further list of Ptolemaic horoscopes in given in Table III C.

SIDEREAL HOROSCOPES[51]

	Vettius Valens	hours from mdnt	Sun ordinal degrees[52]	modern	diff
1	54 oct 29	8	♏ 10	4;18	5;42
3	67 sep 13	6	♍ 25	17;58	7;2
4	69 jul 16	11	♋ 28	21;10	6;50
5	75 jul 19	9 pm[53]	♋ 27;45	24;4	3;39
6	79 mar 16	9 pm	♓ 29	24;7	4;53
7	79 nov 26!	4 pm	♐ 8	3;4	4;56
8	83 may 15	7	♉ 27	21;25	5;35
9	85 nov 20	12	♐ 3	29;19	5;41
10	102 nov 28	2	♐ 8;30	4;55	3;35
11	105 apr 21	10	♈ 29	29;13	-0;13
12	109 jun 2	8	♊ 13	9;17	3;43
14	114 jul 26	6 pm	♌ 5	1;7	3;53
15	114 aug 10	10 pm	♌ 22	15;47	6;13
16	113 sep 10!	3	♍ 22	15;38	6;22
17	115 jun 8	11	♊ 20	14;41	5;19
20	118 nov 26	10	♏ 7	3;20	3;40
21	119 mar 25	8 pm	♈ 7	3;9	3;51
23	120 feb 8	7 pm	♒ 22	18;30	3;30

[51] The dates marked (!) present inconsistencies.
[52] Except where the minutes are given.
[53] Midnight, according to Neugebauer (1975).

29	121 may 18	9 pm	♉ 29	25;36	4;24
30	125 mar 24	8 pm	♈ 6	2;43	3;17
32	127 oct 28	2 pm	♏ 8	3;48	4;12
33	131 jul 10	10	♋ 20	15;18	4;42
34	134 feb 26	10	♓ 9;46	6;38	3;8
35	142 jan 24	4	♒ 6	3;29	2;31
36	151 feb 17	3 pm	♓ 3	27;20	5;40
37	151 nov 23	10 pm	♐ 2;42	0;46	1;56
38	152 jan 8	6	♑ 20	16;58	3;2
41	157 nov 24	8 pm	♐ 6	2;15	3;45

Table III A. List of horoscope dates from Neugebauer (1975), p.181.

		date	hours from mdnt	Sun ordinal degrees[54]	modern	diff
42	L 115,II iii,10,25	115 feb 15	4	♒ 29	k 25;10	3;50
43	L 114,V	114 may 13	22	♉ 25;18	20;54	4;24
44	L 110,III	110 mar 15	19	♓ 25;8	23;37	1;31
45	No 81 P. Lond. 130	81 mar 31	20;36	♈ 14;6	9;13	4;53
46	L 76, CCAG	76 jan 24	6	♒ 8	2;44	5;16
47	L 40, CCAG	40 apr 16	12	♈ 19	13;24	5;36

Table III B. Supplementary list of horoscopes with sidereal longitudes, selected from Neugebauer (1975).

source	Ptolemaic	source	Ptolemaic
P. Oxy 1476[55]	No 260	CCAG	L478
CCAG	L380	CCAG	L479
Marinus	(L412)	CCAG	L486
CCAG	L428	CCAG	L487
CCAG	L440	CCAG	L497
CCAG	L474	Stephanus	L621

Table III C. List of horoscopes calculated from Ptolemy's tables, selected from Neugebauer (1975).

STOBART TABLES AND BERLIN P.8279

There are two Demotic Egyptian Planetary tables, the Stobart Tables, a set of four wooden tablets, and the Berlin Papyrus P.8279, both of which have been edited and analysed by Neugebauer.[56] In both of these we have the date (as the year of the Roman Emperor, month and day) when each of the five planets enters a new zodiacal sign. The Berlin papyrus covers the years -15 Aug 29 to 11 Aug 30, and the Stobart Tables cover altogether the period 71 Aug 30 to 132 Aug 29. Since these lists give only the dates of sign entry, they fall well short of providing the sort of data that we would have from a true ephemeris. Neugebauer, however, recovered enough to convince himself that these tables were based on a calculation of the sidereal longitudes of the planets, and he even attempted to determine the difference between the position and the tropical longitude. By way of illustration we have here in Fig. 15 the calculated and tabulated values for Mars from Stobart Tablet C, for 5000 days from 105 Sept 13.

[54] Except where the minutes are given.
[55] Not in Jones 1999.
[56] Neugebauer 1942 for the first edition and thorough analysis, Neugebauer & Parker 1960-9 for a revision of the edition, and brief additional remarks.

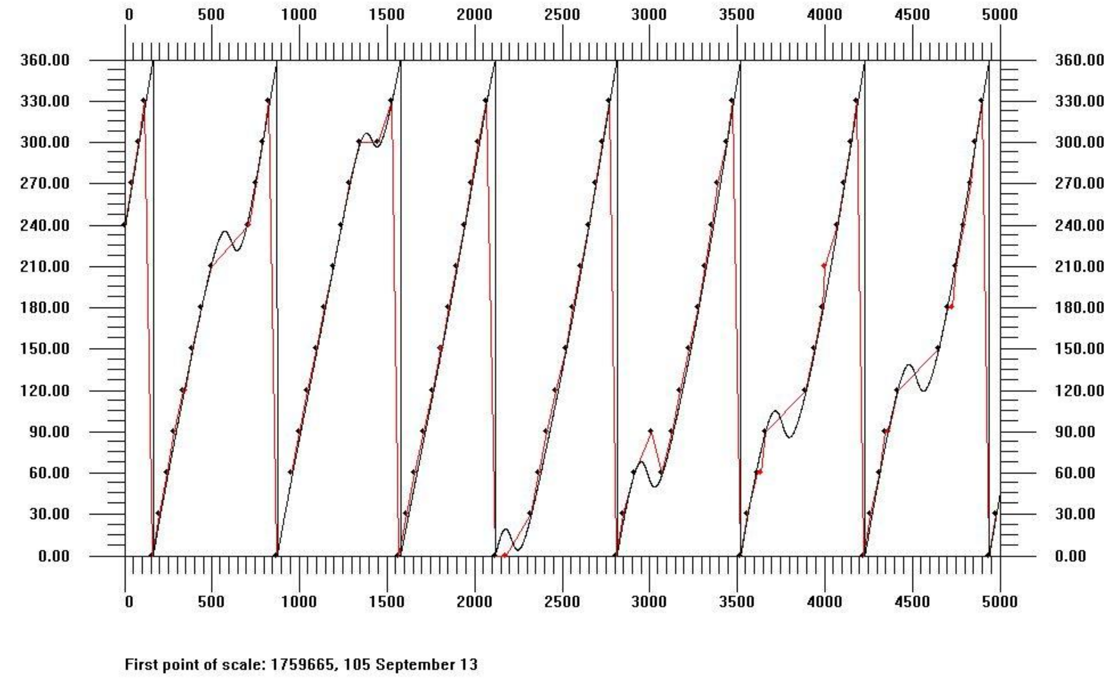

Fig. 18. Longitude of Mars in Stobart Table C: calculated tropical longitude, and the dates of entry into successive signs, shown by the dots.

The excess of the tabulated longitudes over the calculated are, as one can see, quite irregular. The excesses are shown in Fig. 18.

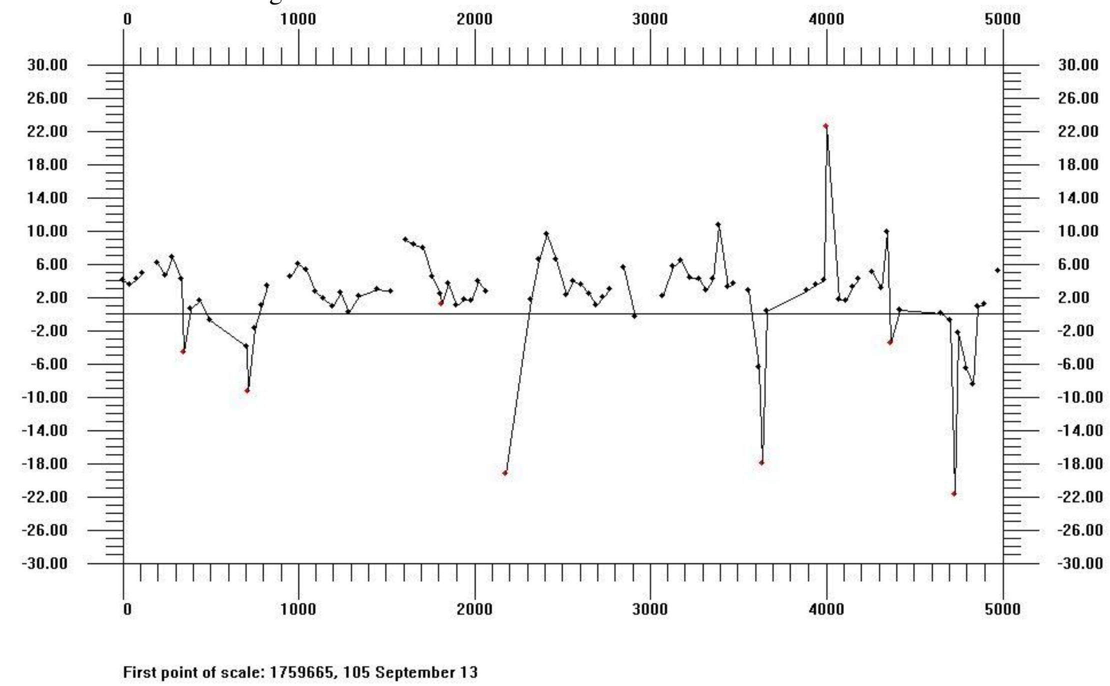

Fig. 19. Longitude of Mars in Stobart Table C: excess of longitude as tabulated over the calculated tropical longitude.

Neugebauer was content to describe this excess, for Mars, as lying in the range 1½° to 2½° , but as one can see from Fig. 19, the actual excess is very widely scattered, between about 0 and 6° ! For these dates the excess by Theon's precession is about 4½°. For the other planets, except for Mercury, the situation is similar, and equally discouraging, if one hoped to find support here for any particular model of precession. Indeed for Mercury the excesses are negative, as Neugebauer reported. This is shown in Fig. 20.

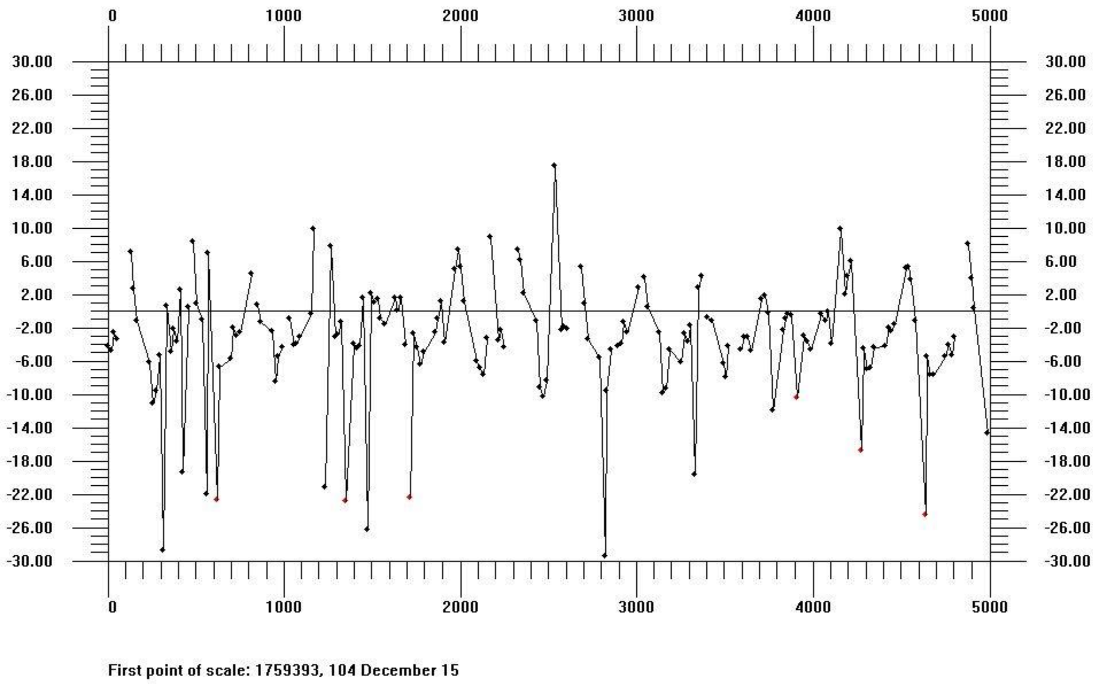

Fig. 20. Longitude of Mercury in Stobart Table C: excess of longitude as tabulated over the calculated tropical longitude.

In conclusion, it must be said that while these planetary tables on the whole support the view that sidereal longitudes were used, there is no clear quantitative result.

PERPETUAL TABLES

Some horoscopes are associated with the 'perpetual tables', attributed to Egyptians, for example P. Lond 130, ll. 12-3: κανόνων αἰωνίων.[57] This is dated CE 81 March 31, a horoscope with coordinates whose excesses lie in the range 5 to 6. The coordinates are plainly sidereal, with excess values lying within the group shown in Fig 17. These perpetual tables are also mentioned by Vettius Valens VI, 1, and (with some disapproval) by Ptolemy in the *Almagest* IX.2 (αἰωνίου κανονοποιίας). [58]

The text accompanying P. Lond 130 is unusually prolix, compared with the other horoscopes from either Vettius Valens, or extracted from passages in the CCAG. One can well suppose that many others were also calculated from these Perpetual Tables.

THE SIDEREAL ECLIPTIC OF INDIAN ASTRONOMY

In his Siddhānta Śiromaṇi Bhāskara II (early 12th century) remarks that at the time of Brahmagupta precession was too small to be considered, whereas in his own time it was larger, and must be allowed for.[59] Here he reminds us in effect that there is a sidereal ecliptic peculiar to Indian astronomical practice, one that continued unchanged from the time of Brahmagupta, and indeed from Āryabhaṭa. It is evident from the deviation curves of the canon of the last named author that precession was negligeable, for the tight node in the curves occurred when the deviations were essentially zero, that is when sidereal and tropical ecliptics coincided. Was that really just a coincidence ? I will argue now that in fact Āryabhaṭa himself created the new sidereal ecliptic, and indeed created in this way a new standard that was to continue not only through Indian astronomy, but in the branch of the Arabic work where they explicitly followed the Indian template, such as that of al-Khwārizmi, and of Zarqala in the

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[57] Neugebauer & van Hoesen 1959, No. 81 (pp. 21-8).
[58] Vettius Valens: (ed.) Kroll 243.8, (ed.) Pingree 232.29; Ptolemy: Heiberg II 211.5.
[59] Siddhānta Śiromaṇi, Grahagole : golabandha, commentary to śl 17-19.

Toledan Tables, and even as late as Copernicus. The fortunes of that sidereal ecliptic were studied by me at length some years ago.[60] What prompted Āryabhaṭa to redefine the sidereal ecliptic ?

In the course of the above discussion of Pingree's article of 1976 we have already noticed that at the epoch of Āryabhaṭa (499 Mar 21) the mean tropical Sun calculated according to the *Almagest* was - 2;44. If on the other hand the sidereal equivalent had been found by adding precession according to Theon, it would be −2;57. If Āryabhaṭa were faced with either of these he would conclude that this erroneous result could be corrected by altering the parameters of the tropical Sun, since he must have recognized from his own observations that the Spring Equinox occurred on March 21.[61] Indeed he set the mean sidereal Sun to be zero at Noon on that day, and one must assume that he intended as well that the sidereal and tropical Sun should be equal at that point. *In so doing he was defining a new sidereal ecliptic*; this sidereal ecliptic would be used henceforth in Indian astronomy. Compared with the sidereal ecliptic implied by Theon's model of precession this redefinition amounts to a shift of roughly 3 degrees. If it were the case, as argued above, that the point opposite the star Spica (α-Vir) had served in effect as the origin for Theon's sidereal ecliptic, then a shift of 3 degrees would shift the origin to the point in the Hipparchian tropical ecliptic opposite to 174- 3 = 171, that is to a point near to - 9 on the Hipparchian scale, or near - 6;20, (ie 353;40) in the Ptolemaic tropical ecliptic.

From the Indian point of view the origin point of the sidereal ecliptic was seen not only as the head of Meṣa (Aries), but as the head of Aśviṇī, the first of the 27 nakṣatra divisions. These nakṣatras are each referenced by junction stars, and the junction star nearest to the head of the first, Aśviṇī, is known to be ζ-Psc which is given either the sidereal longitude 0, in some texts, or in others, 0;10 short of that, near the end of the last nakṣatra Revatī. This identification of the star has been accepted since the beginning of European studies of the Indian texts, and there is no reason to question it, although there are doubts about the identification of some other junction stars. It is listed by Ptolemy in the *Almagest* and the *Handy Tables*, and of course it is known from modern listings, such as the Yale Bright Star Catalogue. From the modern listing we find that its ecliptic longitude is zero in 575, and 359;50 in 565; the latitude is -0;14.[62] According to Ptolemy the longitude is 353;0, with latitude -0;10. It is in fact a double star whose two components are of magnitudes 5.2, 6.3, respectively, but perceived together as a star of 4th magnitude by Ptolemy.[63] Needless to say, such a faint star was hardly likely to have been selected as an origin point, even though it is on the ecliptic; it is in any case in the wrong place in the year 499.

If the position of the Indian sidereal ecliptic is anchored to this junction star of Revatī, but not as that star was to be observed at that time, how was it fixed ? We will now show that there is good reason to believe that the position of the star is taken from the Greek star list.

Now in an earlier research of mine I presented a sidereal alignment that appeared to help to explain this situation, by way of relating the Indian ecliptic to the Greek star coordinates.[64] This is shown in Fig. 22, which illustrates that the Indian origin point is the junction with the ecliptic of a great circle passing through α Ari, β Ari, ζ Psc, and the point $\underline{\alpha}$ opposite to α Vir. Moreover this circle is the horizon for the geographic latitude of 36^{o}, so that α Ari, β Ari, ζ Psc lie on the eastern horizon, while α Vir is setting in the West. It is found that where this circle of alignment intersects the ecliptic the longitude of that point, reckoned from the hypothetical origin of Hipparchian coordinates, is 9;23±0;5.[65] This alignment therefore has the striking property of marking the Indian zero point of the ecliptic, as that point of the ecliptic that rises on the Eastern horizon simultaneously with α Ari, β Ari, ζ Psc and with the setting of α Vir, and all at the very geographical latitude that one associates with Rhodes and Hipparchus.[66]

The interval 9;23 may be taken as the precise difference between the Indian sidereal ecliptic and the Hipparchian ecliptic of -127. According to Ptolemy ζ Psc is at 23^{o} Psc, that is 353;0, and the Hipparchian position is 350;20.[67] Therefore the alignment places ζ Psc in Indian ecliptic at 359;43±0;5. The precessional change from Hipparchus to the time of Āryabhaṭa being 9;23, then over the interval from -127 to 499 the implied rate of precession is 53.96±0.4″ p.a., or 1 degree in 66.7±0.27 years.

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[60] Mercier 1976-7.

[61] By modern calculation on that day the mean Sun is zero at about 6 p.m. on the central Indian meridian.

[62] The calculation is based on the coordinates of ζ Psc in the Yale Bright Star Catalogue, nos. 361, 362.

[63] Magnitude is a logarithmic measure, so that if the brightness is doubled the magnitude is reduced by one.

[64] Mercier (1976-7), Part II, p.33

[65] The tolerance 0;5 allows for the evident rounding of stellar coordinates in the Greek listing, where the longitudes are rounded to the nearest multiple of 0;10.

[66] Although this alignment is precise for the Hipparchian positions of these stars, it is somewhat less so for the true positions of the stars at that time.

[67] Its true position in -127 was 350;15.

Although in the extant text and quotations from his work Āryabhaṭa does not mention precession, we can be quite sure that like his contemporaries he knew the phenomenon. Bhāskara I in his commentary on the Āryabhaṭīya notes that it was known to the Romaka school, who understood the motion to be about 47″ p.a.[68] Versions of precession, here denoted ayanāṃśa in terms of the Indian sidereal ecliptic, are found in later canons, notably the later *Sūryasiddhānta*, according to which the ayanāṃśa vanishes in 499, and with the motion 54″ p.a., which is a more typical motion. This is similar to many other versions of ayanāṃśa to be found in the Indian texts. In Fig. 21, a number of such precession models are shown, largely reproducing the graph presented in the earlier study of mine.[69] Here the curve of ayanāṃśa according to the later *Sūryasiddhānta* passes through the Hipparchian origin at -9;23 in the year -127.

Much has been said above about the model presented by Theon, where the rate was 45″ p.a., and which was surely used with the *Handy Tables*, but there is also a Greek text, in an as yet unedited papyrus, in which sidereal coordinates are combined with a model of precession with a rate of 46″ p.a.[70]

We emphasize again the implication of the above argument, that the position of the star ζ Psc in the list of junction stars of the nakṣatras was known to the Indian scholars according to some version of the Greek star list. The Indian texts place it either at 0 or at 359;50, and the latter agrees as near as matters with that implied by the alignment. *The Indian astronomers knew of this junction star according to a Greek source.* We ought therefore to consider as well the other junction stars, for if they knew one they will have known all the others as well from the Greek list. Can one infer that they knew of the alignment ? That is a difficult question, and certainly takes us beyond the evidence, since we must be clear that so far there is no *direct evidence* in Greek sources either. The exactness however of the alignment, together with the beautiful connection between Greek and Indian coordinates that it lays bare, leaves no doubt in my mind at least that it was really recognized at least by Hipparchus.

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[68] Shukla (1976), Part 2, pp. lviii-ix; commentary on Āryabhaṭīya, kālakriyāpāda i.5, p. 183.
[69] Mercier (1976), Part 2, p. 48.
[70] The edition is being prepared by Jean-Luc Fournet and Anne Tihon.

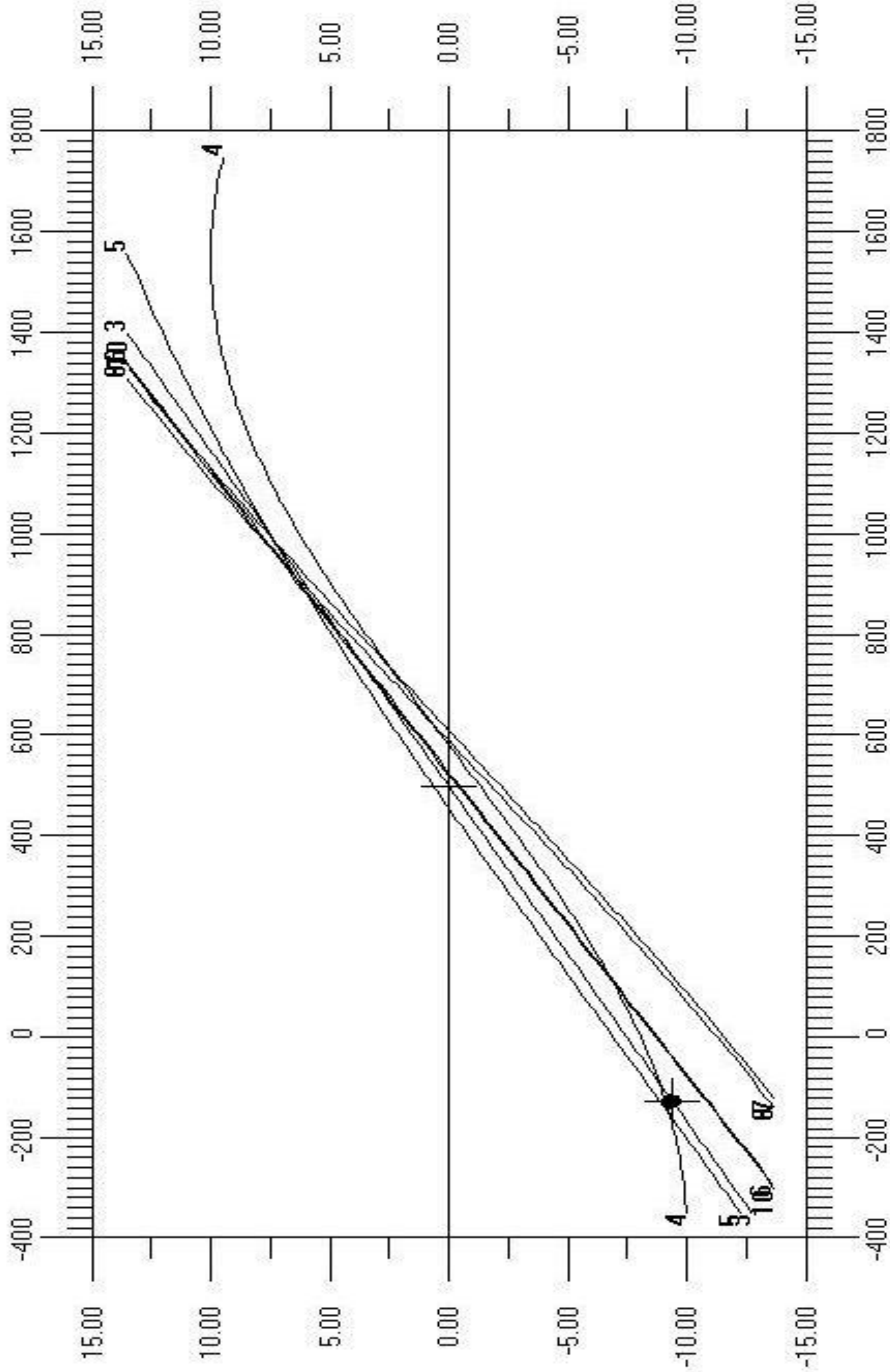

Fig. 21. Models of precession. Legend: 3 Sūryasiddhānta (later), Karaṇa Tilaka, Tantrasaṃgraha, Siddhāntadarpaṇa; 4 Ibn Kammād; 5 Alfonsine; 6 Bhoja, Śrīpati, Bhāskara II, Āmarāja; 7 Parāśara; 8 Mahāsiddhānta; 9 Karaṇa Kalpa; 10 Grahalāghava. The Hipparchian origin is marked by the dot at (-127, -9;23).

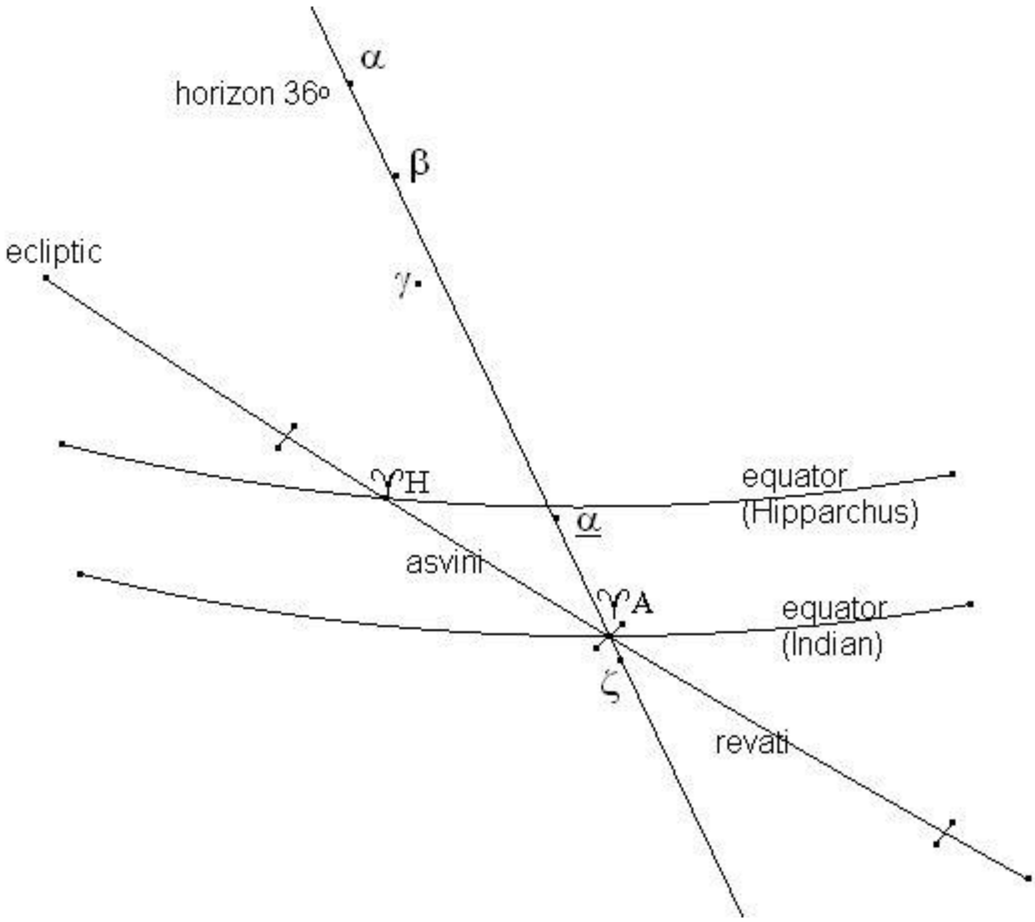

Fig. 22. Positions of stars in α, β Aries and ζ Piscium, and anti-Spica α. The alignment circle through the stars is the horizon for the geographic latitude 36^0, which also determines the Indian Zero point (at the head of Aśvinī near ζ Psc), where this circle intersects the ecliptic. At the time of Hipparchus the celestial equator passes nearly through α, and at the time of Āryabhaṭa the celestial equator passes through the Indian origin, near ζ Psc. The equinoctial points $♈_H$, $♈_A$ are for the epochs of Hipparchus and Āyabhaṭa.

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It is widely assumed that the coordinates of the junction stars are so-called 'polar' longitudes and latitudes, as explained by Burgess.[71] It may well be questioned however whether this is the correct interpretation, and I will argue in another place that the coordinates ought in fact to be taken simply as ecliptic. Moreover it is suggested that the list of junction stars has been corrupted through neglect, because in fact none of the treatises from Āryabhaṭa onwards makes any practical use of the coordinates, apart from the reference to the star at the origin point. In spite of the corruption of the text tradition, there a good half of the entries permit a reasonably clear identification, and are such that the longitudes are derived from Ptolemy's list by adding 6;20 to his longitudes, or 9;0 to the hypothetical longitudes of Hipparchus. Shukla, in his edition of the *Mahā-Bhāskarīya*, claimed that in certain texts, namely the *Mahā-Bhāskarīya*, the *Laghu-Bhāskarīya*, and the *Śiṣyadhīvṛddhida*, the longitudes are ecliptic, while in other texts they are polar, and that 'this explains why there are significant diffeence between them'.[72] These 'differences', however, are slight and unsystematic, and surely are not to be explained in this way.

[71] Burgess, 1960, Ch. VIII.
[72] Shukla, 1960, p. 102.

Conclusion

Billard pursued the problem of establishing the scientific content of the Sanskrit canons by applying to them an approach that differed little from that brought to a wide range of ancient materials by many scholars. What could be more reasonable than a comparison between the ancient data and a modern calculation ? Billard differed from others in that he plotted this quantity 'ancient minus modern' as a function of time, and so discovered the great utility of considering the bundle of such deviations. This provides, as it were, an X-Ray of the canon. This approach has been strangely misunderstood, as if it were some exotic methodology, which could be readily faulted whenever it revealed features which one happened to dislike, such as Pingree's belief in the *Paitāmahasiddhānta*, or Plofker's belief that Parameśwara had observed as successfully as he claimed. But like an X-Ray, the bundle of deviations shows just whatever it shows, evidence about the canon which cannot be denied, but which then has to be interpreted.

REFERENCES

Billard, R.

1971 *L'Astronomie indienne. Investigation des textes Sanskrits et des données numériques,* Publications de l'École française d'Extrême-Orient. Volume LXXXIII : Paris.

1980 Le Romakasiddhanta. Unpublished typescript.

Burckhardt, J.J.

1958 Zwei griechische Ephemeriden. *Osiris* 13: 79-92.

Burgess, E.

1860 Sūryasiddhānta. Translation of the Sûrya-Siddhânta, a text-book of Hindu astronomy, By Rev. Ebenezer Burgess [with commentary by W.D. Whitney], *Journal of the American Oriental Societ*y, vol. VI, 1860; frequent reprints.

Chatterjee, B.

1981 *Dhīvṛddhida Tantraof Lalla with the Commentary of Mallikkārjuna Sūri*, edition and translation by Bina Chatterjee, Indian National Science Academy, Delhi 1981.

Dikshit, S. B.

1981 *English translation of Bharatiya Jyotish Sastra*. Government of India Book Depot. [This is a translation made from the original in Marathi, Pune, 1896]

Dvivedi, V.P. ed.

1912/17 *Jyautish Siddhánta Sangraha*. Two fascicles. Benares 1912, 1917.

Jones, A.

1999 *Astronomical Papyri from Oxyrhynchus (P. Oxy 4133 - 4300a). Memoirs of the American Philosophical Society*. Volume 233.

Jyautish Siddhánta Sangraha – see Dvivedi.

Kennedy, E.S. & B.L. van der Waerden

1963 The World-year of the Persians. *Journal of the American Oriental Society* 83: 315-27.

Mercier, R.

1976-7 Studies in the Medieval Conception of Precession. *AIHS* 26:197-220, 27:33-71.

1987 The meridians of reference of Indian astronomical canons. G. Swarup, A.K. Bag & K.S. Shukla eds., *History of Oriental Astronomy, Proc of IAU Colloquium 91, New Delhi, 13-16 Nov 1985.* Cambridge: University Press: 97-107.

1993 The Date of the Mahāsiddhānta. *Gaṇita-Bhāratī. Bulletin of the Indian Society for History of Mathematics* 15:1-13.

The last three are reproduced in:

2004 *Studies on the Transmission of Medieval Mathematical Astronomy.* Variorum Collected Studies Series CS787. Ashgate Variorum, Ashgate: Aldershot Hants.

2007a 'Astronomical Computation for the History of Indian Astronomy', in *Traditions of Science, Cross-cultural Perspectives. Essays in honour of B.V. Subbarayappa*, edited by Purushottama Bilmoria and Melukote K. Sridhar, Delhi: Munshiram Manoharlal, 2007; pp. 72-82.

2007b The Standard Scheme of the Moon and its Mean Quantities. *Archive for History of Exact Sciences* , **61** 2007, 255-272.

Nau, F.

1910 *Notes d'astronomie Syrienne, J. Asiatique*, sér. 10, t. xvi, 1910 pp. 209-228.

Neugebauer, O.

1942 *Egyptian planetary texts. Transactions of the American Philosophical Society* 32: 209-50, Plates I-XXVII.

1956 An Astronomical Almanac for the year 348/9 (P. Heid. Inv. No. 34). *Historisk-filologiske Neddelelser, Det Kong. Danske Vid. Selskab* 36, nr.4.

1975 A *History of Ancient Mathematical Astronomy*. Springer Verlag: Berlin & New York

Neugebauer, O & R. A. Parker

1960-9 *Egyptian Astronomical Texts* I-III. L. Humphries: London

Neugebauer, O & D. Pingree

1970-1 *The Pañcasiddhāntikā of Varāhamihira*. Det Kongelige Danske Videnskabernes Selskab Historisk-Filosofiske Skrifter 6,1 Parts I, II: Copenhagen.

Neugebauer, O & H.B. van Hoesen

1959 *Greek Horoscopes. Memoirs of the American Philosophical Society*. Vol. 48.

Nobbe, C.F.A.

1845 *Claudii Ptolemaei Geographia*, ed. Carolus Fridericus Augustus Nobbe, Leipzig 1845; repr. Olms Verlag, 1966.

Pingree, D.

1963 Astronomy and Astrology in Indian and Iran', *ISIS* 54: 229-46.

1965 The Persian 'observation' of the Solar apogee in *ca*. CE 450', *Journal of Near Eastern Studies* 24: 334-6.

1968 The *Paitāmahasiddhānta* of the *Viṣṇudharottarapurāṇa*, The Adyar Library Bulletin Vols. XXXI-XXXII, 1967-8.

1970a On the classification of Indian planetary tables. *Journal for the History of Astronomy* 1: 95-108.

1970b Ārybhaṭa II. *Dictionary of Scientific Biography*, Vol. 1: 309-10.

1973a Indian influence on Sasanian and early Islamic astronomy and astrology', *The Journal of Oriental Research (Madras)*: 34-5 (1964-66, published 1973): 118-126.

1973b The Greek influence on early Islamic Mathematical Astronomy. *Journal of the American Oriental Society* 93:32- 43.

1976 The recovery of early Greek astronomy from India. *Journal for History of Astronomy* 7: 109-23.

1978 History of mathematical astronomy in India. *Dictionary of Scientific Biography*, Volume 15, Supplement I.

1980 reply to van der Waerden (1980). *Journal for the History of Astronomy* 11: 58-62.

1990 The Purāṇas and Jyotiḥśāstra astronomy', *Journal of the American Oriental Society* 110: 274-80.

1992 On the Date of the Mahāsiddhānta of the second Āryabhaṭa. *Gaṇita-Bhāratī, Bulletin of the Indian Society for History of Mathematics* 14: 55-6.

Plofker, K.

2009 *Mathematics in India,* by Kim Plofker, Princeton University Press, 2009.

Renou, L. & J. Filliozat

1947 *L'Inde classique. Manuel des Études indiennes*. Payot: Paris .

Sarma, K.V.

1995 The Pañcasiddhāntikā of Varāhamihira: a puzzling problem in its presently available text. *Adyar Library Bulletin* 59 : 211-7.

Śāstri, Bāpū Deva

1929 The *Siddhānta Śiromaṇi, A treatise on Astronomy by Bhāskarāchārya*, edited by Pandit Bāpū Deva Śāstri, Benares, 1929

Sastry, T.S.K.

1978 The epoch of the Romaka Siddhānta in the Pañcasiddhāntikā, and the epoch longitudes of the Sun and Moon in the Vāsiṣṭha-Pauliśa. *IJHS* 13 : 151-8; [reproduced in Sastry (1989)]

1989 *Collected Papers on Jyotisha*. Kendriya Sanskrit Vidyapeetha, Tirupati, 1989. Rashtiya Sanskrit Samsthan, New Delhi, Tirupati Series No. 52 : 188-200.

Sastry, T.S.K. & K.V. Sarma

1993 *Pañcasiddhāntikā of Varāhamihira with translation and notes* by T.S. Kuppanna Sastry, critically edited with introduction and appendices by K.V. Sarma, P.P.S.T. Foundation [Patriotic and People Oriented Science and Technology Foundation], Adyar, Madras, 1993. P.P.S.T. Science Series No. 1.

Shukla, K. S.

1960 *Mahā-Bhāskarīya*, edited and translated into English by Kripa Shankar Shukla, Lucknow: Lucknow University, 1960.

1976 *Āryabhaṭīya of Āryabhaṭa with the commentary of Bhāskara I and Someśvara*, critically edited by Kripa Shankar Shukla, Three Parts, Delhi: Indian National Science Academy, 1976.

Stückelberger and Grasshoff

2006 *Klaudios Ptolemaios Handbuch der Geographie*, ed. Alfred Stückelberger and Gerd Grasshoff, Two parts, Basel, 2006.

Suter. H. ed.

1914 *Die astronomischen tafeln des Muḥammed ibn Mūsā al-Khwārizmi*, A. Bjørnbo, R. Besthorn, ed. H. Suter, Copenhagen. 1914.

Tihon, A.

1978 *Le "Petit Commentaire" de Théon d'Alexandrie aux Tables Faciles de Ptolémée. Histoire du texte, édition critique, traduction*. Vatican Studi e Testi 282.

Thibault, G. & M.S. Dvivedī

1889 *The Pañchasiddhāntkā, the astronomical work of Varāha Mihira*, the text edited with an original commentary in Sanskrit and an English translation and introduction by G. Thibault and Mahāmahopādhyāya Sudhākara Dvivedī, Chokhamba Sanskrit Series vol. LXVIII : Varanasi.

van der Waerden, B. L.

1980 Two treatises on Indian astronomy. *Journal for the History of Astronomy* 11 : 50-8.

1988a On the Romaka-Siddhānta. *Archive for History of Exact Sciences*, 38: 1-11.

1988b A Summary of Roger Billard's *L'Astronomie indienne*. *Gaṇita-Bhāratī*, *Bulletin of the Indian Society for History of Mathematics* 10: 21-30.