I - ARTICLES

STUDIES IN THE MEDIEVAL CONCEPTION OF PRECESSION

(Part I)

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1. *Introduction*

The phenomenon of precession presents the astronomer with a problem of relative motion. Pre-Copernican astronomers however had at best only an intuitive grasp of relative motion, and certainly did not possess sufficient maturity of understanding of kinematics to be able to analyse such a problem in terms of the addition of velocities. Instead they would discover some geometric locus entailed by the constraints of the model representing the physical situation.

The ancient value of the rate of precession, 1° per 100 years, which we owe to Ptolemy, and which was probably used by Hipparchos, was derived by them apparently by a direct study of stellar longitudes. In fact however, both Hipparchos and Babylonian astronomers were in possession of luni-solar data, partly at least of Babylonian origin as Kugler showed, which entailed a much more accurate value for the rate of precession, but which could only have been exploited by an analysis involving the addition of velocities, or something very much like it. From the Almagest (iv. 2), the 'exeligmos'[1] of 126007 $\frac{1}{24}$ days contained 4267 synodic months, and 4612 sidereal lunar revolutions less 7.5°. The synodic and sidereal velocities are therefore 12.1907472763 and 13.1763469568 degrees per day, the difference of which is the sidereal velocity of the Sun, 0.9855996805. When compared with the Hipparchan tropical year, which gives a tropical velocity $360/365.24\dot{6} = 0.9856352784$, we obtain the rate of precession, the velocity of the equinoctial point against the stars, of 0.0000355979, or 46.807″ p.a. All this was pointed out by Biot and Sédillot (12, p. 83; 52, p. 437), and seems obvious enough, and yet Delambre missed it in spite of his extensive study of Hipparchos's discovery, and Manitius in a footnote to his translation of the Almagest (72, vol. 2, p. 196 n. b) clearly makes the mistake of thinking that the exeligmos entails a value of the tropical year. These same considerations led Kugler and Schnabel to feel certain that the Babylonians had discovered precession, since values of the sidereal and tropical years are implicit in Babylonian records, although it became

[1] This long period, which is approximately the product of the 19-year seasonal period with the 18 year eclipse period, must have been discovered in Babylonian eclipse records, and of course the Babylonians must have known of it. I have calculated its mean value, taking the dates of lunar eclipses from Oppolzer (67) and the more exact times from Goldstine (37), and with 44 pairs of lunar eclipses from (A.D. –519, A.D. –174) to (A.D. –471, A.D. –126), the mean value is 126007 days, 75.7 minutes; the number of surplus minutes ranges from 33 to 116. The interval 7.5° is the least accurate element in this description, for it should be nearer to 6.5°.

clear eventually that they did not (58). It is therefore clear that ancient astronomers, both Babylonian and Greek, were not able to make these calculations. A value for the sidereal year (365.2568126) is often presented as 'Ptolemy's', but we must understand that he never quoted any such figure, nor does it seem likely that he could have calculated it exactly in terms of the tropical year and the rate of precession[2]. This is not due to any lack of mathematical skill as such, but to an ignorance of kinematics.

He did not even calculate the sidereal month, nor make any use of the interval 7.5°. The sidereal month occurs only in Babylonian and Indian astronomy, the Babylonian value agreeing exactly with that derived from the exeligmos using 7.5°, as Kugler showed (45, p. 46). The Indian values agree nearly with that derived if one uses 6.5°.

An intuitive approach to the problem is found first of all in Medieval astronomy. Thābit ibn Qurra, for example, calculates the tropical year from values of the sidereal year and a rate of precession (62, p. 284), and it is interesting to translate his argument into a formula expressed in terms of velocities. He writes,

> If you, however, wish to know the time of the solar year, defined by the return to a point of the zodiac, by means of the known motion of the fixed stars, then you can easily find this, provided you know the point from which the sun begins to move in the zodiac; and you should know how much belongs to the sun from the motion of the fixed stars during the year from its mean motion at the point from where it began its motion. And then let us know the length of time in which the sun traverses this amount with its mean motion. Then you subtract this time from the solar year, whose beginning lies at a point of its circle, until it returns to the same.

If v_p, v_s, v_t are the rates of change of stellar longitude, sidereal solar longitude and tropical solar longitude (so that we ought to have $v_t = v_s + v_p$) then Thābit says that we calculate the movement of precession in one sidereal year, which is $v_p(360/v_s)$, calculate the time occupied by the sun in moving through this angle, $360v_p/v_s^2$, and subtract from the sidereal year, $360/v_s$, to obtain the tropical year $360/v_t$. This implies the relation

$$v_t = \frac{v_s^2}{v_s - v_p} = v_s + v_p$$

which only agrees well with the correct relation, because $v_p \ll v_s$.

Abraham ibn Ezra (12th C) (89, p. 82–3) gives an argument to relate the two year lengths, an argument which is repeated uncritically by Roger Bacon (8, p. 16). According to them, one associates with the tropical year a rotation in proportion to the excess of 365 days, so that with Ptolemy's value 365.24$\dot{6}$ we have 0.24$\dot{6}$ × 360 = 88;48. From the rate of precession 1° per 100 years one obtains another rotation 360/100 = 3;36, and upon adding these we have 92;24 as the rotation of the

[2] Aaboe (1) has presented calculations corresponding to this discussion of the exeligmos, but directly in terms of the Babylonian parameters, which entail a sidereal year equal to 12;22,8 synodic months (365.2506 days). Babylonian tables of equinoctial dates depend however on a tropical year equal to 12;22,6,10 synodic months. See § 9.2.

sidereal year, 92;24/360 = 0.25$\dot{6}$. This argument, which is much weaker than Thābit's, is both dimensional and kinematic nonsense, and illustrates well the desperate weakness of this vital aspect of medieval astronomy.

Copernicus calculates the rate of change of the tropical solar longitude quite correctly by adding velocities (de Revolutionibus iii. 14), and it is quite clear that by his time there had been achieved, presumably by his immediate predecessors, a clear operational grasp of velocity. So far, it would seem, historians have documented little if any of this development.

There is a curious feature of the Alfonsine tables which may offer a clue to the growing awareness of the addition of velocities. For, the mean tropical rate of the sun's motion as used in these Tables is given quite precisely[3] by the formula

$$\frac{360}{365.25}\left(1 + \frac{1}{49000}\right) = 0;59,8,19,37,19,13,56,13,\ldots$$

which allows one to believe that this rate was seen as the sum of two elementary rates, both, unfortunately, rather artificial. In any case there is a strict adherence within the Alfonsine tables to a standardized daily rate of motion and to a standardized measure of time measured in natural days and sexagesimal fractions of a day. This represents surely a distinct conceptual advance, and not just a saving of paper!

In calculating the sidereal year itself medieval tables in the Babylonian tradition come closer to the notion of the addition of velocities. Indeed the mean motion of the Sun against the stars was calculated correctly from the lunar rates in the Indian works such as the Sūrya Siddhānta, and in the Khwārizmian and Toledan Tables[4]. In the Sūrya Siddhānta (i. 35; 19, p. 25) we are given the number of lunar sidereal and synodic revolutions in a mahāyuga of 4320000 years as 57753336 and 53433336 = 57753336 — 4320000, respectively, and this relation entails the same result as if one subtracted velocities. Nevertheless it is important to recognize the conceptual difference from the kinematic approach.

In the Khwārizmian Tables the mean sidereal motion of the Sun is 0.985603528464 (calculated from Table 4 in Suter's edition according to which the displacement is 304;15,25 in 720 Arab years; 82, p. 115), while the sidereal motion of the moon is 13.176355401234 (61, p. 92). This entails a synodic rate 12.190751872770, which is equivalent to the larger of the two synodic months, 29;31,50,5,45, used in the table of lunar conjunctions (61, p. 109). The same feature is found in the Toledan Tables (87, p. 80).

It is perhaps strange therefore, but nevertheless true, that precession was not treated in this way, although again the Indian texts seem to verge upon this approach. For example, in the Siddhānta Śiromaṇi (§ 7 of this paper), which takes precession as a continuous rotation, not as trepidation, we are told that the

[3] The full result quoted is found in a version of the tables in Br. Mus. MS Royal 12. D. VI fol. 7r (c. 1400). The printed tables give the first seven figures only.

[4] It must be understood that the sidereal year can never be the result of direct solar observation, but is purely derivative from the lunar periods: the question is whether this derivation is correct or not.

equinoctial point makes 199669 revolutions in a kalpa of 4320000000 years, which implies that 4320199669 tropical years = 4320000000 sidereal years. For some reason however this clear inference is never made, nor is the tropical year ever mentioned.

The notion of velocity clearly requires in turn some notion of a universal time as a dimension of the universe independent of spatial coordinates. The tendency in pre-Copernican astronomy had generally been to regard time as a complex patchwork of incommensurable periods, rather than as a formally continuous mathematical quantity, and the periods in question were expected to have some celestial significance. At first sight nothing would seem more 'natural' than the natural day as a unit of time, but in the Babylonian tradition this seems to have been regarded as a terrestrial rather than a celestial unit. They preferred instead to use complicated calendars geared to celestial phenomena, in particular phenomena plotted against the stars in the night sky[5].

The Babylonian tables present one with a self-consistent account of the night sky, not as unchanging, to be sure, but one in which the changes of planetary position are correlated amongst themselves, rather than with the Sun. The sense of time is therefore rather weak, since one experiences time primarily through solar phenomena. Greek astronomy shows all the opposite characteristics, and seems to be primarily concerned to record the precise solar time of events. It followed that positions of the planets, Sun and Moon were measured from the equinox of date, and that their motion was expressed in terms of tropical and synodic periods.

Clearly a knowledge of precession was necessary in order to relate these two astronomical traditions, and this presents apparently a greater problem to the Babylonian tradition, for there all results calculated from the table need to be corrected for precession in order to obtain a prediction in terms of local time. In the Greek tradition on the other hand, because planetary positions are given directly in terms of the equinox of date, and so in local time, it would seem that an understanding of precession is less important. However, the problem in this tradition is then that of making basic observations in the night sky, making use of the stars for primary measurements, and so the stellar position must be given in relation to the equinox. The Babylonian approach makes more sense on the whole from the point of view of experimental method, and the results were in fact more accurate in this tradition in the Middle Ages. In particular the sidereal year was known with greater accuracy than the tropical year.

In order to find a satisfactory articulation between these two astronomical traditions people had to develop a notion of relative motion, which in turn required a notion of time which embraced phenomena in the nightly as well as the daily sky. From this deep problem of selfconsistency there emerged gradually the notions of absolute universal time, and kinematics, which form the backbone of the Copernican synthesis.

We begin the discussion in this paper with an example of a medieval criticism of one of the most influential models of precession, that of Thābit ibn Qurra.

[5] Even in modern astronomy one has to distinguish between 'ephemeris time' the continuous variable appropriate to Newtonian dynamical theory, and 'universal time', which is a mere counting of days, the difference arising from the fact that the rate of spin of the earth on its axis is slowing down. In modern physics and cosmology the notion of time is once again in the melting pot, largely as a result of Einstein's theories, and it is easier for us, than for Kant for example, to appreciate that time as used in Copernican and Newtonian theory is a specific scientific hypothesis.

2. *The Toledan Tables in A.D. 1300*

When Guillaume de St. Cloud, a leading figure in Parisian astronomy towards the end of the thirteenth century, wished to compare and corroborate astronomical tables known to him at that time, he proceeded to determine the precise time of the Spring equinox in the year 1290, and he deduced from this the 'movement of the eighth sphere' to be 10;13. This is recounted by Guillaume in his work 'Almanach Guillelmi de Sancto Clodoaldo ad annos 20 incipit anno 1292', BN MS 7281 fol. 141a–144b. Duhem (31, vol. 4, 16–17) and Littré (51) give full accounts of Guillaume's observations. To be more exact, Duhem and Littré repeat clearly what Guillaume reports, but do not explain just how it is that one could ever determine a movement of the eighth sphere, that is a movement of precession, from the date of the equinox. The explanation, however, turns out to be simple enough. The tables which Guillaume had with him included those of Ptolemy, Alexandria, Tolosa and Toledo. The Toledan tables, unlike those of Ptolemy, were based on the sidereal year (taken as 6,5;15,23,29,24 . . . days), and have this feature in common with the Khwārizmian and Indian tables which precede them in a tradition associated, one might say, with astrology and with Babylonian methods.

The calculation of the Sun's position then, according to the Toledan tables, would give its position with respect to a point in the ecliptic which is fixed in relation to the stars, and this would be the correct longitude of the Sun with respect to the equinoctial point only at one particular date. For all tables in this 'astrological' tradition, that date is located somewhere in the range A.D. 450–650, and the point of the ecliptic which serves as a zero of longitude agreed in right ascension with the end point of the lunar mansion known as Baṭn al-ḥūt in Arabic, Revatī in Sanskrit, and K'uei (奎) in Chinese, determined by the junction stars, β Andromedae, ζ Piscium and η Andromedae, respectively.

We may verify Guillaume's figure of 10;13 by making a calculation of the Sun's position for the point in time which he determines as that of the equinox, 16^h after noon of Sunday 12th March 1290 (Julian date 2192301.$\dot{6}$, ignoring the correction due to the difference in longitude between Greenwich and Paris). We simply substitute this date into the Toledan Tables and we ought to get—10;13, so that the movement of the eighth sphere in question is the precessional movement from that date when the equinox was located at a point near ζ Piscium.

The Toledan Tables have been surveyed by Zinner (100) and Toomer (87), but they do not give quite enough information to enable us to carry out the complete calculation of the Sun's true position, for in particular, they do not give the complete table of the equation of the Sun. Moreover in calculating the mean motion of the Sun, one might try to work with a reduction of the table to a modern formula, but this would only obscure certain irregularities which are inherent in the structure of the tables, due for example, to the irregular distribution of the eleven Arab leap years in each cycle of thirty.

Therefore we have made the calculation using one particular copy of these tables, included in Cam. Univ. Lib. MS Ii. 3.3 fol. 117–216 (A.D. 1276). We first calculate the number of days elapsed from 31st Dec. 1 B.C., given by the table on fol. 153a,

1288 years	2, 10, 40, 42 days
1	6, 5
Jan.	31
Feb.	28
12 days of Mar.	12
total =	2, 10, 47, 58

From this one subtracts 1, 3, 3, 35 which is the number of days elapsed from 31st Dec. 1 B.C. up to 14th July A.D. 622, leaving 1, 7, 44, 23 days. Making use of the next table, fol. 153b, this interval is distributed over Arab years and months,

660 years	1, 4, 58, 2 days
28	2, 45, 22
2 months	59
total =	1, 7, 44, 23

On fol. 181a, 181b we have the table of the longitude of the mean Sun in terms of the Arab calendar

660 years	7^s 19;48,54
28	1^s 29;11,33
2 months	1^s 28;9,3
16 hours	0;39,25
total =	11^s 17;48,55

Note that $1^s = 30°$, so the longitude of the mean Sun is 347°48′55″ in the more familiar notation.

In order to calculate the equation of the Sun, which must be added to the longitude of the mean Sun, we first find the anomaly, that is the distance of the mean Sun from the apogee, which is given in the Canons of the tables, fol. 126a, as 2^s 18;50. The anomaly is therefore 8^s 29;58,55. The tables of the equation are given following those of the mean motion and we must make a linear interpolation from the entries.

8^s 29 equation = 1;59,10
9^s 0 ,, = 1;59,4.

Interpolation gives 1;59,4 and so the true Sun has the longitude 11^s 19;47,59. This falls short of the full circle by 10; 12, 1, which is to be compared with Guillaume's figure of 10;13. These are close enough to permit us to claim that he was calculating his result by using the Toledan tables.

Guillaume goes on to refer to that particular theory of precession, Thābit ibn Qurra's model of trepidation, which seems to have been generally associated with the Toledan tables. If this model gave a correct account, and if Guillaume's observation of the date of the equinox were correct, then the precession could be calculated as 10;13 (or 10;12), whereas Guillaume's calculation from Thābit's tables gave him 9;23. Because of this discrepancy he rejected, it seems, not only the theory of trepidation, but the Toledan tables as well. These tables were in fact based on an exceptionally accurate value of the sidereal year, 6,5;15,23,29,24, the modern value being 6,5;15,22,54. After this time Latin astronomers generally used the Alfonsine tables which were based on the tropical year, and it was not until Copernicus succeeded in formulating a new synthesis between this 'astrological' tradition and the Ptolomaic tradition, that astronomers returned to the use of tables based on the sidereal year.

Turning now to Guillaume's calculation of the figure 9;23, we find that the tables in Thābit's work *de motu octauae sphaerae*, which are given also in the MS mentioned above (fol. 88b–89a), require us to calculate first the *motus octauae sphaerae*, which is given by the rule,

$$1;34,2 + (2;34,58)\ (\text{Arab year}/30).$$

The tables give the values of this expression for intervals of 30 years and 1 year, so that we have

660 years	*motus* =	1^s 28;23,20
28		2;24,38
1/6		0; 0,51
	total	2^s 0;48,49

One now turns to the second table for the precession itself, here called *equatio diversitatis longitudinis capitis arietis*, whose values are given very roughly by the rule 10;45 sin *(motus)*. The entries are in steps of 5°,

motus	*equatio*
60	9;17,44
65	9;43,53

A linear interpolation gives 9;21,59 corresponding to the *motus* 60;48,49. While this differs slightly from Guillaume's result, it is near enough, like the former calculation of 10;13, to permit us to claim to have followed his own calculation.

The table of values of the *equatio* is given in full later (Table III).

The precise time of the Spring Equinox can be determined in accordance with modern parameters by using Tuckerman's Tables (88), from which we find 2192301.73[6], i.e., 17;50 hours after noon of 12th March. Guillaume's error, then, is about as small as one could expect with non-telescopic observations.

[6] These Tables give the solar longitude at 10 day intervals to an accuracy of 0.01°. At each of the years 1288–1292 a four point inverse interpolation is made using dates either side of the zero in longitude, and then a linear regression curve is fitted to these five interpolated equinoctial

Although Guillaume's observation of the time of the equinox is very accurate, it is useful to calculate the deviation he would have found had he employed the correct time of 0.73 day after noon of 12th March. In the additional 0.0613 day the movement of the Sun is 0;3,38, and this would reduce the deviation to about 10;9.

3. The Toledan and Khwārizmian Tables in the sixth century

The deviation in the calculation of the Sun's position according to the Toledan Tables arises, as explained above, from the difference between the true tropical year and the value of the sidereal year implied in the Toledan Solar Tables, and in view of this we can work out the rate of change of the error. Rough calculations indicate that the deviation is zero at some point in the sixth century, and we make use of this in arranging a more precise calculation.

The Toledan Tables of the mean Sun are all based on the assumption that the motion in 30 Arab years, 10631 days, is 10478;0,21, and this entails a daily rate 0.9856 0867 5885. On the other hand, making use of the modern formula for the length of the tropical year (7, p. 537)

$$365.24219878 - 0.00000614 \text{ T days,}$$

where T is the time elapsed since A.D. 1900 in units of 36525 days, we find that the mean rate of motion of the Sun, in tropical longitude, for the year A.D. 900, situated midway between A.D. 1300 and A.D. 500, is 0.9856 4716 9778. The rate of change of the error is therefore 0.00003849 3893, equivalent to a "precession" of 50.616″ p.a. The deviation in 1290 (2192302) is about 10;9, and so it is zero in A.D. 564.

One ought to consider a small correction due to the increase in the equation of the Sun at the time of the true equinox, for this increases from 1;56,36 to 1;59,4 (as the anomaly decreases from 280° to 270°). This decreases very slightly the rate of change of the error to 0.00003838, which however puts back the date of the zero deviation by less than one year.

This may be confirmed by calculating directly the movement of the eighth sphere in the years A.D. 560–4. The equinoctial dates are calculated by a four-point inverse interpolation from Tuckerman's Tables, to which is fitted a linear regression as for the years 1288–1292 in § 2.

It is interesting to compare corresponding results for the Khwārizmian Tables, which have been edited by Suter (82), and translated with a commentary by Neugebauer (61). These also are based on the sidereal year, although all the parameters are slightly different as seen in the table below. In these Tables the Sun is referred to the meridian of Arin, which is the ancient site of Ujjain, situated 75;46 East of Greenwich (11, p. 53). However Neugebauer gives two illustrations in his commentary (pp. 94, 111) which indicate that in this Spanish recension at least, the real

dates. This procedure is expected to give dates reliable in the second decimal place, perhaps with an accuracy ± 0.002.

meridian for the Solar Tables is around Baghdad, 45° East of Greenwich. The date of the Spring Equinox referred to Baghdad time would be 0.125 days less than for Toledo.

The relevant parameters for the Toledan and Khwārizmian Tables are next given, followed by the dates of Spring Equinox, and the movement of the eighth sphere in these years.

SUN	Khwārizmian	Toledan
mean rate	0.9856 0352 8464	0.9856 0867 5885
radix	113;25,47	113;41,11
apogee	77;55	77;50
equation at equinox	2;11,46	1;56,32

MOVEMENT OF EIGHTH SPHERE

A.D.	Equinox	Khwārizmian	Toledan
560	1925674.82_{40}	—0;2,10	—0;2,41
561	1926040.06_{71}	—0;1,15	—0;1,54
562	1926405.31_{03}	—0;0,21	—0;1,7
563	1926770.55_{35}	0;0,33	—0;0,19
564	1927135.79_{66}	0;1,27	0;0,28

It is seen that the movement changes sign between 562–3 for the Khwārizmian Tables, and between 563–4 for the Toledan Tables. It is very remarkable that these two tables, which differ in every one of the four solar parameters, point nevertheless to the same year when the sidereal longitude of the Sun equals its tropical longitude.

Of course these calculations which we have made to find the deviation at the true equinox, were not available to Medieval astronomers, and there is no way in which they might have inferred from the internal structure of these solar tables that they were correct in A.D. 563. It is also unreasonable to suppose that the date of the equinox for that year was known with sufficient accuracy to enable the solar tables to have been adjusted then, or at a later time, so that they were without error in A.D. 563. In any case it is not clear in what way medieval users were concerned to interpret what we have called the deviation.

In fact one can only make a simpler inference, viz., that as part of some convention of measurement, the solar longitudes were measured from some point fixed in relation to the stars, since after all, the tables are based on the sidereal year. While we can easily find the position of the equinoctial point for A.D. 563, it is again not the case that the medieval astronomer could have calculated the same position. The modern determination is made using the formula (4, p. 530)

$$p = 50.2564'' + 0.0222'' \text{ T p.a.}$$

where T is the time elapsed since A.D. 1900, expressed in units of 36525 days. Between A.D. 563 and A.D. 1900 the mean rate is therefore 50.108″ p.a., and the longitude of the equinoctial point is 18;36,34 (A.D. 1900). It is of historical interest to compare this with the longitude of ζ Piscium, which is found by the use of P. V. Neugebauer's Table (38) to be 18;26 (A.D. 1900). Thus the equinoctial point, to which longitudes were referred in the Toledan and Khwārizmian tables, is about 0;10 East of ζ Piscium. This, it must be emphasized, is not necessarily where the astronomer who used these Tables believed the origin of longitude to be situated. Nevertheless a model of precession described in the Sūrya Siddhānta does indeed take the point in question to be just there. This has been known since the researches of Colebrooke into the Indian accounts of the lunar mansions (nakṣatra), (22, Vol. 2, pp. 283–303).

3.1 Indian and Sassanian sources behind al-Khwārizmī

This question of the zero point of sidereal longitude will occupy much of this paper, and in § 5 we explain the historical origin of the zero point as a product of the technique of star mapping in early Hellenistic astronomy.

Although the Ancient and Medieval astronomers knew where to locate the zero point in relation to the stars, there were considerable variations in the choice of date on which the equinoctial point would coincide with it. I know of only one explicit medieval statement of this date, and otherwise the historian is left to infer it from models of precession and trepidation, apart from the study of the Tables such as we have just given. This solitary reference is provided by Isaac Israeli in his elaborate treatise on the Jewish calendar, Yĕsōd ha ʿŌlām (43, p. ix of the first part of the translation), who says that the moving and fixed zodiacs coincided around 750 years before the time of composition of his work, which can be firmly placed in 1310; therefore coincidence occurred in around 560. This is not only the only explicit statement of the date of coincidence, it is also surprisingly accurate. In none of the models of precession to be surveyed in the following pages is the implicit date of coincidence so accurately given.

There is an interesting problem here regarding the date 562/3. The authors of both the Khwārizmian and Toledan Tables must have had it explicitly in mind, for in no other way could it have been arranged that the two solar Tables could agree. One has to decide between a calculation of this date found by making use of some model of precession related to the Greek star catalogue, and actual observations made very close to A.D. 560. No model of precession known to us would give the equinoctial point 0;10 East of ζ Pisc at exactly this date, and certainly none of the pre-Islamic models come near to so doing. This leaves one to suppose there were observations of Indian or Sassanian origin made sufficiently carefully around A.D. 560.

In his recent important study of Indian material, Billard (11) has studied the deviations of the mean longitudes of Sun, Moon and planets compared with the corresponding modern formulae. He actually calculates not the exact date at which any particular deviation vanishes, but rather the date at which the sum of squares of selected deviations is minimum. In the case of the Brāhmasphuṭasiddhānta (A.D. 628) these dates for various selections of deviations range over the sixth century, while in particular the date of minimum value for the sum of squared deviations for Sun, Moon, lunar apse and node, and all the planets save Mercury, is A.D. 562.8 (11, p. 117). From Billard's point of view this date might therefore be regarded as the date which represents overall the origin of the sidereal coordinates. This date is virtually the same as that found above in the analysis of the Toledan and Khwārizmian Tables, and reinforces the conclusion already argued by Suter, Burckhardt and Neugebauer, that the Khwārizmian Tables descend in part from the Brāhmasphuṭasiddhānta, taken to be that Indian system known to the Arabs as the Sindhind. Indeed the parameters relating to the mean motion are virtually the same in these two works. (See: 61, pp. 93, 131 and the references to Burckhardt on p. 235; 82, p. 32; 71, pp. 16, 28.)

However one ought to calculate the date on which the true, not the mean, Sun is exact at the Spring Equinox. The procedure for calculating the equation of the Sun is explained (if that is the right word) in the Brāhmasphuṭasiddhānta (17, ii. 20 seq.), where the basic sinusoidal formulation is inflated by empirical modifications. These do not seem to be of importance in the present calculation, so we take the equation to be given by the rule,

$$\sin \eta = -\frac{41}{1080} \sin (\mathcal{L} - \tilde{\omega})$$

where $\mathcal{L}$ = longitude of the mean Sun and $\tilde{\omega}$ = longitude of the apogee. The date sought may be shown to be A.D. 580, when at the Spring Equinox 1932979.76$_5$,

$$\begin{aligned} \tilde{\omega} &= 77.9072 \\ \left.\begin{aligned} \mathcal{L} &= 357.8418 \\ \eta &= 2.1415 \end{aligned}\right\} & \ \mathcal{L} + \eta = -0.0167 \end{aligned}$$

The value of η is obtained by a self consistent solution of the equation for η, when $\mathcal{L} = 360 - \eta$, and when the values of sin given in the text are interpolated; one is given the sin for multiples of 3;45:

$$\sin 3;45 = \frac{214}{3270} \quad \sin 78;45 = \frac{3207}{3270} \quad \sin 82;30 = \frac{3242}{3270}$$

(15, ii. 5). The procedure of obtaining $\mathcal{L}$ and $\tilde{\omega}$ is straightforward and one may refer to Billard's useful summary of the formulae.

It is now clear that for the true position of the Sun the Khwārizmian Tables do not derive from the Brāhmasphuṭasiddhānta. Another source must be sought for the explanation of the year 562/3, and this turns out to be the Sassanian Tables known as the Zīj al-Shāh, associated with the reforms of Khusrau Anūshirwān (reigned 431–579). These Tables are no longer extant, and our information concerning them derives from various Arabic sources: the Fihrist of ibn al-Nadīm (33, p. 578), the Qānūn'l-Masʿūdī of al-Bīrūnī (14, p. 1473) and the summary of Māshā' allāh's astrological history by ibn Hibintā (71, passim). Modern studies are due mainly to Taqīzādeh (84), Kennedy (44), Pingree (71) and van der Waerden (97). For our purposes we need the passage in al-Bīrūnī's work to which attention was first drawn by Taqīzādeh (84, p. 321). This passage gives the date A.D. 556 when Khusrau convened Persian astronomers, and is the principal source associating the Zīj al-Shāh, or at least its reform, with Khusrau specifically. However a closer analysis of the passage enables us to draw much deeper conclusions. The passage is as follows, taken from al-Bīrūnī's account of the astrological system known as the 'Thousands' *(hazarāt)*, (13, pp. 1473–4)[7].

> In their opinion the seven planets and the two nodes (Head and Tail) follow one another with a constant number of years for each; we call this number *fardāriyāt*[1]. They agree that in the twenty-fifth year of Anūshirwān[2], four years of the *fardāriyāt* of Jupiter had elapsed, and there remained eight[3]. Then the *fardāriyāt* of Mercury, 13 years; then Saturn, 11; then the Tail, 2; then Mars, 7; Venus, ☿ and the Sun for 10 and the Moon 9 and the Head 3[4], then again Jupiter around the Zodiac in relation to its exaltations, during the space of 75 years.
>
> We mentioned the above date (twenty-fifth year of Anūshirwān) because at that time the astronomers of Persia met to correct the Zīj al-Shahrīyārān known as al-Shāh, and they recorded the measures of times and stations (al-naubat wa'l-mīnāhā). They knew from the position of Jupiter at the end of the month Ābān[5] that three *hazar* had passed and of the fourth, 851 years. Thus if we discard 75-year cycles 51 times, there remain 26 years[6], which begins with the *fardāriyāt* of the Sun and ends with the first four years of the *fardāriyāt* of Jupiter. From this point to the beginning of the reign of Yazdagird there are 76 years: 23 of Anūshirwān, 12 of Hormuz, 37 of Parvīz, 4 of Shīruya[7]. Thus at the beginning of the reign of Yazdagird 5 years had elapsed of the *fardāriyāt* of Jupiter.

Notes. 1) *fardāriyāt*: for this other related technical terms of astrological history, see Pingree. 2) twenty-fifth year of Anūshirwān: A.D. 556. 3) the *fardāriyāt* of Jupiter ran from 552 to 564. 4) of all these *fardāriyāt*, only that of Jupiter has any astronomical significance, since we have then the sidereal period.5) end of Ābān = early March[8]. 6) $3851 = 51 \times 75 + 26$. 7) Khusrau Anūshirwān, 531–579; Hormuz IV, 579–591; Khusrau Parvīz, 591–628; Shīruya (and others), 628–632; Yezdagird III, 632–651.

The planet Jupiter was manifestly the key to this astrological system of *fardāriyāt*, as also to the reform of the Zīj al-Shāh. When the planet was observed in Ābān 556 it was visible during much of the night, but at the termini of the *fardāriyāt*, in 552 and 564, it was in conjunction with the Sun almost exactly at the moment of the Spring Equinox. Nowhere else in the sequence of *fardāriyāt* of Jupiter or the other 'stars' is there any clear astronomical phenomenon at the termini. A fuller discussion of the behaviour of Jupiter will be given later, after Thābit's model of trepidation has been studied. Here we conclude that the true historical explanation of the use of the year A.D. 564 as a 'canonical' point stems from this Sassanian reform, which happened to take place just when the equinoctial point coincided with the sidereal zero point (see § 5), although in the Brāhmasphuṭasiddhānta the same year is relevant in an average sense. These considerations show that the Khwārizmian Tables used the mean motions of the Brāhmasphuṭasiddhānta, but

[7] I am most indebted to Mr H. R. Farran, a research student of this department for the translation.

[8] The New Year, Nauroz, 1st Farvardin, occurred on 16/6/632 at the accession of Yazdagird III, and it is believed that al-Bīrūnī used the vague year, not the intercalated Zoroastrian year; Ābān is the eighth month counting from Farvardīn.

adopted an equation (and perhaps a method of calculating an equation) exactly in line with the Sassanian reform. Suter (82, p. 33) has quoted ibnu'l-Qifṭī who in fact reports just this, 'al-Khwārizmī in his Tables relied on the mean positions of the Sindhind, but departed from it regarding the equations and the declination (of the ecliptic); he established his equations according to the method of the Persians, and the declination of the Sun according to Ptolemy's method'. (See also van der Waerden, 96, 97.)

4. Thābit ibn Qurra's model of precession

The theory of trepidation may derive from the desire in various culture areas, to allow for the phenomenon of precession by way of reconstructing in post-Hipparchan times, the appearance of the night sky as it was in 'antiquity', so that ancient religious or astrological interpretations could be conserved. To this extent we are not dealing with pure Astronomy, but with 'astrological history', a recurrent feature in the history of Astronomy. Apart from the well known remarks of Rhaeticus in the Narratio Prima, who gives a historical interpretation of Copernican trepidation, one may also cite recent studies of Abū Ma'shar and Māshā'allāh (71), and Longomontanus (55). We find sometimes that the Eastern limit of the movement of trepidation is a position in the ecliptic of some phenomenon which had marked the beginning of the year; for example, in this way in the Indian models one appears to encounter the heliacal rising of α Arietis and the heliacal setting of the Pleiades.

Thābit ibn Qurra's[9] contribution to the theory of precession is available only in a Latin version entitled *de Motu Octavae Sphaerae* which has been edited by Carmody (83), and translated with a commentary by Neugebauer (62). The geometry of the model has been the subject also of recent papers by Goldstein (36), Dobrzycki (29) and North (64). In spite of these studies there is still much that needs to be established about the model in order to clarify its position in the history of pre-Copernican astronomy. Thābit's contribution was dismissed by Delambre with the famous judgement 'Son malheureaux système de la trepidation infecta les tables astronomiques jusqu'a Tycho.' (26, p. 73), a remark echoed by many later historians. This is harsh and unjustified, for Thābit's model was no more improbable, or lacking in self-consistency than any other feature of pre-Copernican astronomy, and provided moreover better agreement with the stellar longitudes known to Hipparchos, Ptolemy and Arab astronomers than did any other model earlier than that developed by Copernicus.

[9] Thābit is a product of the extremely important culture of Ḥarrān in Northern Mesopotamia (lat. N 36;51, long E 39;1) where one had not only a very conservative maintainance of pagan worship, in the form of a Moon-cult, but also a centre for the diffusion of Greek and Syriac learning. There is an extensive article on the subject in the Fihrist, and Dodge (33, p. 745) in his translation of this work gives further references, particularly to the monumental study of Chwolson, Die Ssabier und der Ssabismus (21), where one will find a biography of Thābit, and a list of his very numerous writings. Ḥarrān was situated 100 km due north of al-Raqqa, the site of al-Battānī's observations.

4.1 *The geometry*

The familiar construction of great circles representing the equator and ecliptic is supplemented for present purposes by a second ecliptic circle. These are called the fixed and moving ecliptics respectively (see Fig. 1), the moving ecliptic being

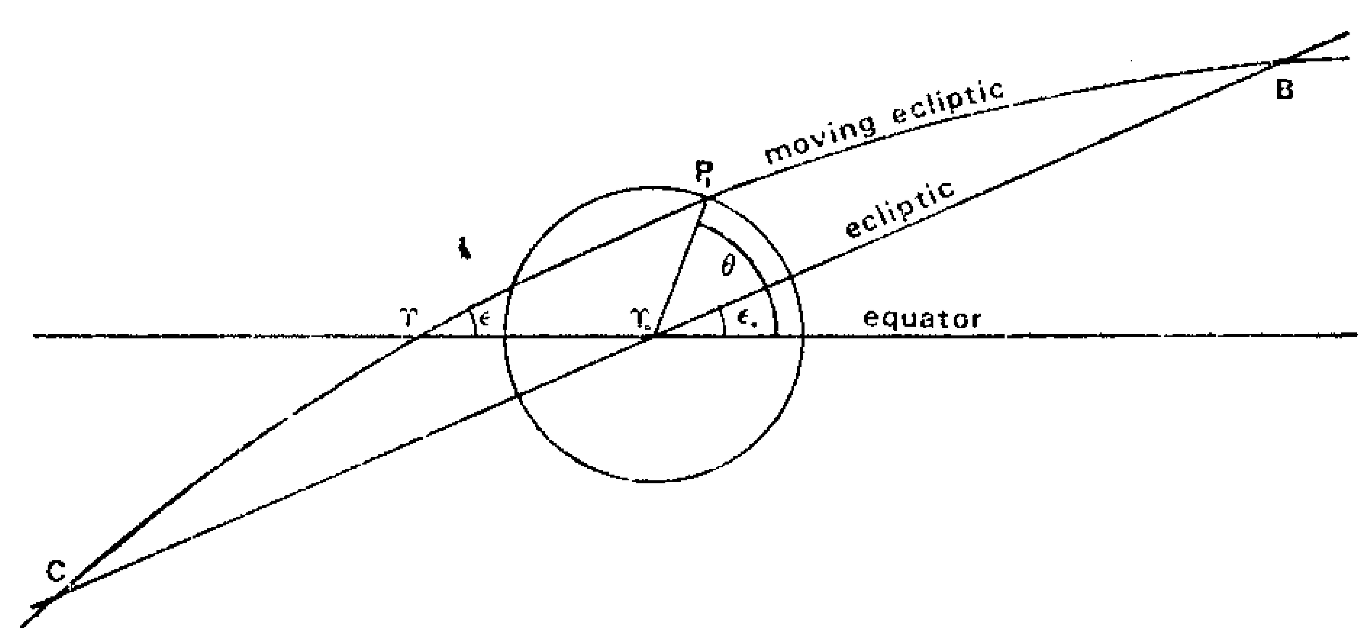

Fig. 1. Thābit's model of trepidation. View from the outdie of the sphere showing the small circle at the Spring Equinox, and the three great circles.

in fact fixed in relation to the fixed stars in the eighth sphere. To complete the construction one proceeds as follows. Taking as centres the two points of intersection of equator and fixed ecliptic, one describes two small circles lying in the sphere, and having a radius r in angular measure. The moving ecliptic intersects the small circles in diametrically opposed points P_1 and P_2, P_1 (Caput Arietis) being on the circle centred on the spring equinox. The moving ecliptic is determined finally, for each position of P_1, by constraining the intersections B and C, where the two ecliptics meet, so that $P_1B = 90°$. An alternative constraint has been proposed by North (64) and Goldstein (36) in which the point B is kept a distance 90° from the centre of the small circle Υ_0, but for reasons which will be given later the present choice is to be preferred. The position of P_1 on the circle is located by the angle θ, called the motus, measured from the equator, as shown in the diagram. Thābit's treatise leaves some uncertainty perhaps as to whether the motus should be measured from this point, or from the point where the small circle intersects the ecliptic. We follow Neugebauer's reading (62, p. 296)[10], measuring θ from the equator. P_1 moves at a constant rate around the circle in the direction of increasing θ, which is indeed clockwise as seen by an earthbound observer (counter-clockwise in the case of P_2). This rotation causes a seesaw motion of the moving ecliptic, and in turn a motion of the eighth sphere, the sphere of the fixed stars, for the moving ecliptic, and also the two points P_1 and P_2 are fixed in that sphere. The point of intersection of the equator and the moving

[10] Thābit remarks (83, p. 103, line 17) that the movement of accession or recession is slowest when $\theta = 90°$, but this does not directly settle the question because we have to decide also whether he measures this movement by the arc $P_1\Upsilon$ along the ecliptic, or by the arc $\Upsilon_0\Upsilon$ along the equator, for these choices are consistent respectively with a measurement of θ from the equator or the ecliptic. Naturally the movement $P_1\Upsilon$ along the ecliptic corresponds best with our notion of precession, and is the choice made here. However Carmody takes the other view. See also the remarks made later, on the early analysis by Pedro Nuñez.

ecliptic denoted Υ in Fig. 1 is the spring equinoctial point, which moves to-and-fro in relation to P_1, the distance between them reducing to zero when $\theta = 0°$, or $\theta = 180°$. The distance $P_1\Upsilon$ is measured as an angle $E(\theta)$, a certain function of θ, the angle E being positive when $0 \leqslant \theta \leqslant 180°$, negative otherwise. In the Latin text the angle E is called *equatio diversitatis longitudinis capitis arietis ab equatore diei*, while the angle θ is called *motus accessionis et recessionis 8e sphere*. When the *motus* increases from —90° to 90°, Υ moves Westward relative to the stars, as it is actually observed to do, and we have *accessio* (Arabic 'iqbāl, from the root qabala, to come, arrive), and the end point $\theta = 90°$ is the limit of accession (Arabic al-nihāyat al-'iqbāl). In the next two quadrants $90° < \theta < 270°$, moves Eastward, in disagreement with observation, and we have *recessio* (Arabic 'idbār, from root dabara, to turn away). *Accessio* and *recessio* therefore describe the movement of Υ when it is respectively with and against the daily motion of the stars.

Thābit, like most writers, tends to use the terms *accessio* and *recessio* together without distinguishing which is which. However he does say (83, p. 104, line 17) 'Huic uero motui predicto accidit ut inuenitur tarde diuersitatis et uelocis diuersitatis, et illud est quoniam cum caput Arietis fuerit super 90 ab equatore diei in duabus partibus septentrionis et meridiei, accessio erit tunc tarde diuersitatis; et cum caput Arietis fuerit propinquum sectioni circuli parui cum equatore diei erit accessio tunc uelocis motus; et similiter inuenerunt consideratores.' In the following line he applies these observations to the work of Ptolemy and others, and so clearly intends that *accessio* is the observed motion of the equinoctial point. Another example helps to fill in the interpretation. The diagram of a model of trepidation reproduced by Hartner from a work of Quṭb al-Dīn al-Shīrāzī (40, p. 628) marks the Western limit of the movement of Υ as al-nihāyat al-'iqbāl. Many other references to trepidation found in the widely scattered literature bear out this interpretation of *accessio*.

We shall see later that, in Thābit's model, the angle E vanishes near the end of the year A.D. 604. The fixed and moving ecliptics do not coincide then, but at a later date during Thābit's lifetime[11], A.D. 870, at which time $E = r$.

There emerges a complex picture in which the 'moving ecliptic' with its points P_1 and P_2 are fixed in relation to the stars, while the equator, the fixed ecliptic, and the small circles are continually moving. However for the medieval observer it is the equinoctial point Υ and the equator which are fixed, while the stars and the ecliptic move.

In Thābit's model the moving ecliptic, which is the real ecliptic of observational astronomy, makes an angle ε with the equator, and this angle of obliquity varies between narrow limits as P_1 traverses the circle. The angle between the fixed ecliptic and the equator, ε_0, lies between these limits, and when $\theta = \varepsilon_0$ we have $\varepsilon(\varepsilon_0) = \varepsilon_0$, so that the two ecliptics coincide, and this happens, as noted above, during A.D. 870.

[11] It has been argued by Duhem (31, vol. 2, 257) that this model of trepidation is not actually due to Thābit, but rather to Zarqalu. If that were the case if is not likely that the date A.D. 870 would figure so prominently in the working of the model. Millás Vallicrosa (90) has attempted to refute Duhem's views.

This model of trepidation is worked out in detail in the appendix, where the functions $E(\theta)$ and $\varepsilon(\theta)$ are found to be given by the expressions,

$$\sin E_I = \sin\theta \tan r \left\{\frac{1 + \tan^2 r \cos^2(\theta - \varepsilon_0)}{\tan^2 r \sin^2(\theta - \varepsilon_0) + [\sin\varepsilon_0 + \tan^2 r \cos(\theta - \varepsilon_0)\sin\theta]^2}\right\}^{1/2} \quad (1a)$$

$$\cos\varepsilon_I = \frac{\cos r \cos\varepsilon_0 + \tan r \sin r \cos(\theta - \varepsilon_0)\cos\theta}{[1 + \tan^2 r \cos^2(\theta - \varepsilon_0)]^{1/2}} \quad (1b)$$

When the moving ecliptic is constrained in the alternative way so that $B\Upsilon_0 = 90°$ then E and ε are equal to the functions $E_{II}(\theta)$ and $\varepsilon_{II}(\theta)$ where

$$\sin E_{II} = \sin\theta \sin r \left\{\frac{1 + \tan^2 r \sin^2(\theta - \varepsilon_0)}{\sin^2\varepsilon_0 + \tan^2 r \sin^2(\theta - \varepsilon_0)}\right\}^{1/2} \quad (2a)$$

$$\cos\varepsilon_{II} = \frac{\cos\varepsilon_0}{[1 + \tan^2 r \sin^2(\theta - \varepsilon_0)]^{1/2}} \quad (2b)$$

These four expressions have been calculated for the range of θ, —90(10)90, and the results are given in Table I, and the graph of E_I is shown in Fig. 6. The values of the basic parameters are $r = 4;18,43$ and $\varepsilon_0 = 23;33$, both of which are taken from Thābit's text. Values of E and ε outside this range may be found by using the relations $E(\theta + \pi) = -E(\theta)$ and $\varepsilon(\theta + \pi) = \varepsilon(\theta)$ which apply to both the equations (1) and (2). Note that the actual maximum of E occurs not at $\theta = 90°$, but at $\theta \doteqdot 88;7,46$, when $E \doteqdot 10;40,25$. The entries in Table I for E_I and ε_I agree with Dobrzycki's results (29, p. 14).

In the Latin translations of Thābit's work the quantity E is tabulated for the range 0(5)90, and one is also given values of a quantity called *equatio diametri dimidii circuli parvi*, or in one MS at least (Cam. Univ. Lib. Mm. 3.11 fol. 105a) *distantia caput arietis ab equatore diei latitudo*. Its interpretation has been discussed by Neugebauer (62, p. 298) and following him we take this quantity, here denoted by δ, to be the distance from P_1 to the equator (see Fig. 1). Therefore

$$\sin\delta = \sin E \sin\varepsilon. \quad (3)$$

In Table II we give Thābit's values of E, and ε. In the third column are given the corresponding values of the obliquity, which are calculated from equation (3), taking values of E and ε from the first two columns.

The point P_1, *caput arietis*, as seen from the point of view of the equator and the instantaneous equinox Υ has polar coordinates E, ε. The locus of this point is nearly a straight arc of length about 21° set at an angle to the equator of about 23°. Looked at closely however this locus is seen to be a very narrow closed curve intersecting itself once. This structure is shown enlarged in the graph in Fig. 2. The curve for the alternative model is drawn there also, and this is seen to be a more open figure of eight.

The coordinates E and ε taken from Thābit's data are also plotted in Fig. 2. These values lie very scattered about the exact curves, and only fairly negative conclusions might be drawn from a comparison. One could not really claim that

these points lie particularly close to either curve, although there is at least a tendency for ε to increase with E, as if some crude attempt had been made to allow for the small change in obliquity.

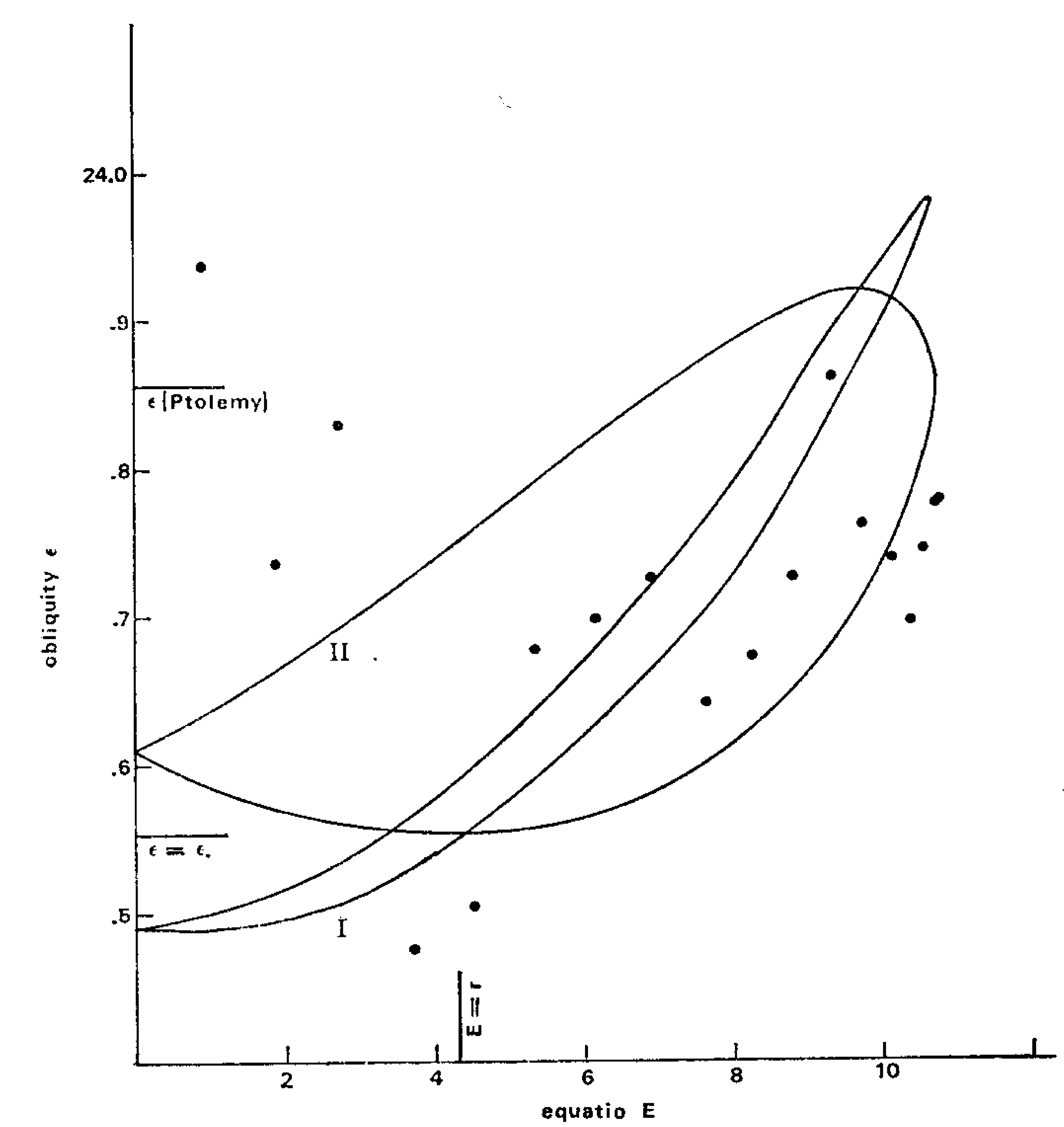

Fig. 2. Graph showing the movement of caput Arietis, with E, ε as Cartesian coordinates. Legend: I, exact calculation with preferred constraint ($P_1B = 90°$); II, exact calculation with alternative constraint ($\Upsilon_0 B = 90°$). The points are based on Thābit's tables. As θ increases each curve is traced counterclockwise beginning at $E = 0$ when $\theta = 0$. Only the range ... ordinate.

The maximum value of E is 10;45, (according to Thābit) and people have sometimes been inclined to argue that the points might have been derived from a Medieval table of sines, so that E would be written 10;45 sin θ. There is surely no question of this interpretation of Thābit's date, although the analogous equation of the eighth sphere in the Alfonsine model is given with fair accuracy by 9 sin θ. Equations (1a) and (2a) yield the approximation

$$\sin E \doteqdot \frac{\tan r}{\sin\varepsilon_0}\sin\theta,\ \varepsilon \doteqdot \varepsilon_0$$

but there would be no historical justification in attempting to relate ($\tan r/\sin\varepsilon_0$) to the figure 10;45. Nor is there any good reason to regard this maximum value of E as a number of fundamental importance in the genesis of the theory, for this value is not related in any particular way to the other values of E.

The arc joining Υ to Υ_0, denoted by c, is calculated also in the appendix, where the preferred constraint is assumed. The formula obtained there is identical to that derived for this quantity by both Delambre (26, p. 278–281) when he develops Pedro Nuñez' clear argument (66, p. 304–7), and by Carmody (83, p. 95), all of whom use the preferred constraint. They assume that the *motus* is measured in relation to the ecliptic rather than the equator, by an angle φ say, where $\varphi = \theta - \varepsilon_0$. The formula for $c(\varphi)$ is thus

$$c(\varphi) = \frac{\cot\varphi \sin(\varphi + \varepsilon_0)}{\sin r \cos r} - \cot r \cos(\varphi + \varepsilon_0).$$

The values of $c(\varphi)$, $-90(10)90$, are given in Table I and it is clear that these values differ by at most four minutes from the corresponding values of $E(\theta)$. Nuñez states clearly that Thābit's equatio is the arc we have denoted by E, and he develops an argument to show that it differs insensibly from c. Carmody makes the mistake of assuming that c itself is Thābit's *equatio*.

4.2 The choice of constraint

In this section I offer reasons for preferring the constraint $P_1B = 90°$ rather than $B\Upsilon_0 = 90°$. In Thābit's text we read (83, p. 104)

> Et ipsius recessio a eo est secundum quantitatem medietatis diametri circuli parvi, nisi quod semper caput Capricorni et caput Cancri sunt coniuncta orbi decliui fixo non recedentia ab eo, et mouenter antecedentes et retrocedentes secundum quantitatem diametri circuli parui.

That is to say, the point B in Fig. 1, which is situated on the fixed ecliptic, is caput Cancri which situated 90° from caput Arietis, the point P_1. Neugebauer (62, p. 290) takes the same choice of constraint.

We next refer to an account[12] of this model given by Peurbach in *Novae Theoricae Planetarum*. This occurs in the last chapter of the work, following an account of Alfonsine trepidation. Peurbach writes (70, fol. 37b)

> Ubicunque etiam sectio harum eelipticarum fiat ipsam necesse est a principio Arietis & Librae mobilium per quartam circuli magni distare.

This is perfectly explicit. Peurbach goes on to refer to the variable angle of obliquity (fol. 38b):

> Unde maxima declinatio eclipticae mobilis ab equatore variabilis est, maior quandoque declinatione eclipticae fixae, quandoque minore eadem, quandoque sibi aequalis. Tunc enim aequalis est illi cum mobilis sub fixae superficie fuerit, maior uero in sitibus contactum. Unde eam Ptolomeus 23;51,20 reperit. Minor autem dum caput Arietis mobilis in sectione aequatoris & parui circuli fuerit.

[12] This particular section of Peurbach's chapter was reprinted as an appendix to the Theorica Planetarum of Gerard of Cremona in the 1480 edition (Bologna), and was mistakenly regarded as the work of Thābit or Gerard by some people including Nallino (56, vol. 1, xxxvi, n. 1), who later corrected his mistake (56, vol. 2, xvi). It is safe to say that Thābit's work was not printed during the Renaissance.

As is clear from Fig. 2, the angle ε indeed takes values both above and below ε_0 with the preferred constraint, but not with the alternative. One can see also that $\varepsilon < \varepsilon_0$ when $E \leqslant r$, in agreement with the last sentence quoted.

Amongst modern writers on this subject, only Dobrzycki has discussed the point and he is in agreement with me. Duhem (31, vol. 2, p. 242), Carmody (83, p. 95) and Neugebauer (62, p. 290) also seem to agree, whereas Goldstein (36) and North (64) take the alternative constraint.

Unexpectedly the alternative constraint would seem to be appropriate to the Alfonsine model, at least in the way in which the model is interpreted by Peurbach.

4.3 The movement of the poles and solsticial points of the eighth sphere

When the preferred constraint is used the pole of the eighth sphere describes a figure of eight curve, while the point B, *caput Cancri*, moves to and fro on the fixed ecliptic. The equations describing this figure of eight are derived in the appendix, where it is shown that

$$\begin{aligned} \xi &\doteqdot -\tfrac{1}{2} r^2 \sin 2(\theta - \varepsilon_0) \\ \zeta &\doteqdot -r \sin(\theta - \varepsilon_0) \end{aligned} \qquad (4)$$

where ξ and ζ are coordinates on the surface of the sphere measured from the pole of the fixed ecliptic in the direction of the summer solstice and spring equinox, respectively. The axis of the curve is the great circle through Υ_0 and the pole of the fixed ecliptic. Peurbach does not make any reference to this curve, although he mentions a similar curve in his discussion of the Alfonsine trepidation, which I will discuss presently.

There may be an illustration of this curve in a curious addition to the diagram of the solar eccentric in a MS of the *Theorica Planetarum* of Gerard of Cremona (Cam. Univ. Lib. Ii. 1.27 fol. 171b), a MS which ends with a note of its author Thomas de Wyndele, A.D. 1424. The figure of eight is drawn over the centres of the Zodiac and solar eccentric. This addition is essentially irrelevant to the text of the Theorica, although there is brief reference to trepidation at the end of this work. It may have been used to illustrate some contemporary discussion. The axis of the figure is at right angles to that of our curve.

When the alternative constraint is used it is not the pole of the eighth sphere, but its caput Cancri, which describes a figure of eight, identical in form to that given by eqn (4), and with its axis coinciding with the fixed ecliptic. The pole of the eighth sphere moves to and fro on a straight arc. This feature is appropriate to the interpretation of Alfonsine trepidation provided by Peurbach. In his diagram, fol. 35a, he shows the figure of eight drawn in this way, and writes (34b),

> Capita uero Cancri & Capricorni octauae sphaerae quasi figuras conoydales habentes pro basi lineas curuas utrinque a capitibus Cancri & Capricorni nouae peragere necesse est. Unde & quandoque praecedent ea, quandoque uero sequentur, quandoque autem coniunguntur, coniunguntur enim caput Cancri octauae & caput Cancri nonae dum caput Arietis octauae fuerit in maxima latitudine ab ecliptica nona & centre circulorum transeunte. Poli autem eclipticae octauae improprie dicti poli quandoque accedunt ad polos eclipticae nonae, quandoque sunt sub eis quandoque uera ab eisdem remouen-

tur, talis tamen accessus & recessus semper est super circulo magno per polos Zodiaci nonae & centre circulorum paruorum eunte.

It is important to be clear that the locus of the pole of the eighth sphere does not completely determine the movement of the stars relative to the equinox, for one must also specify the rotation of that sphere on its axis, and the rotation is given precisely by the equation E.

Some medieval references to trepidation certainly exhibit some confusion concerning the movement of the pole and its relation to the stellar longitude. Abraham bar Ḥiyya writes for example (56, vol. 1, p. 302; 91, p. 119),

> sapientes Indiae, omnes terrae Romanorum incolas, et veteres e sapientibus Chaldaeorum, hanc tantum de stellis fixis opinionem habuisse, stellas non totum Caelum percurrere, sed octo dumtaxat gradibus zodiaci ante et retro, in orientem et in occidentem, moveri. Causam huius motus esse polum eclipticae, qui in parvo circula, (cuius diametrus) 8° zodiaci, in orientem et occidentem circumvolveretur. Illum polum circulum (totum) 1600 annis perficere; ob (hunc) poli ambitum in circulo viros doctos quosdam imaginatos esse stellas fixas totum Caelum ambire, quia arcanum illud non apparet iis manifestum et ignorarent revolutionem poli eclipticae, qua stellae fixae ab occasu ad ortum per 800 annos moverentur, et postea ad locum pristinum, in occasum versus, denuo restituerentur post 1600 annos perfectos.

Here then is an allusion to a model which we associate with Theon's account, but now bar Ḥiyya seems to imagine that the change in longitude of the star entails a displacement of the pole around a small circle, which is certainly not the case.

Bar Ḥiyya's account finds an echo in the diagram from a work of Quṭb al-Dīn al-Shīrāzī (1236–1311), K. al-tuḥfa al-shāhīya fī'l-hai'a, which Hartner reproduced (40, p. 628), for there we see a small circular locus of the pole of the eighth sphere, drawn together with several positions of the moving ecliptic, to show the range of movement of the equinox. The stars as such are not shown, and from this diagram alone we cannot discover how their longitude would change, although Quṭb al-Dīn says that the longitude change also has an amplitude of 8°. This could be the case only if a complex rotation were given to the sphere, partly cancelling the displacement produced by the oscillation of the pole.

4.4 The motus formula

Thābit's tract is followed by tables giving not only the *equatio*, which we have discussed, but also the values of the motus θ expressed in terms of Arab years. The dependence is expressed very simply by the rule

$$\theta = 1;34,2 + (2;34,58) \times (\text{Arab years}/30).$$

We take the Arab year as 354 $\frac{11}{30}$ days, counted from 14 July A.D. 622 (1948438), so that the formula may be written

$$\theta = (t - 1941987)\,\frac{360}{1481800}$$

where t is the time expressed as a Julian date. Thus the period of the motion is 1481800 days, or 4056.947 Julian years, and the angle θ vanishes when $t = 1941987 =$ 14 Nov. A.D. 604.

If we assume that the numbers 1;34,2 and 2;34,58 are obtained by truncating the results of some calculation, so that the 'true' values might be as high as 1;34,3 and 2;34,59 respectively, then we can attach to the period, and to the date when θ = 0, corresponding uncertainties. Therefore the period lies in the range 1481641 to 1481800, while the date when θ = 0 lies in the range $1941987.14\,^{-1.1}_{+0.7}$

The date when $\theta = \varepsilon_0 = 23;33$ is found to be $t = 2038944$, 29 April A.D. 870. Thābit's own dates are A.D. 826/7–901.

4.5 The situation of Thābit's model in relation to the fixed stars

In the discussion of the Toledan and Khwārizmian tables it emerged clearly that the solar longitude provided by the tables was to be measured from a point fixed in relation to the stars, a point in fact situated according to our calculation, 10′ East of ζ Piscium.

Thābit's model is situated in relation to the fixed stars in such a way that the 'moving ecliptic' AP_1B is fixed there as well as the point P_1 itself. From the modern point of view then the small circle is traced by Υ_0 moving around P_1, while the equator is carried by Υ_0 and the point which is diametrically opposed.

In all previous studies, since Delambre, of this well known model there is no mention of the question, where is P_1 situated in relation to the stars? The answer is now fairly clear, of course: since the 'Indian' tables in use in the Middle Ages with which Thābit's and other models of trepidation were used referred longitudes to ζ Piscium, or to a point very near it, *we must take P_1 as that point.*

The precise determination of this point requires some discussion, however. We do not really know how the Medieval astronomer made primary determinations of his parameters, or in what way he actually exercised his skill as an observer. It is not clear whether he simply inherited the stellar reference point along with the Khwārizmian tables, and built upon this in some way, or whether he was capable of primary stellar observations in this connection.

We therefore turn to a comparison of Thābit's *equatio* of the eighth sphere with the longitude of ζ Piscium recorded by, or attributed to various astronomers, in order to gain a further insight into the precise location of the zero of longitude. The value of the *equatio* is found by simple interpolation of the received values (Table II). Where t is the Julian Date, λ is the longitude of ζ Piscium and E is Thābit's *equatio* for this date.

To begin with we notice that Thābit's longitudes at the times of Hipparchos and Ptolemy differ by 2;38,1, and this is sufficiently close to the correct value 2;40 to enable us to conclude that his model was designed specially to provide a variable rate of precession which would accommodate these two authorities, as well as the later Arabic observations. It is not the case that Thābit introduced a 'sinuous' function for the *equatio* simply in order to make the model kinematically 'reasonable'.

The close agreement between the *equatio* and the longitudes of Hipparchos, Ptolemy and al-Ṣūfī seems at first to indicate that Thābit intended ζ itself as the

		t	λ	E
Hipparchos	28/6/—128(1)	1674485	—9;40(4)	—9;43,52
Menelaos	9/2/98(2)	1756892	—7;25(5)	—7;36,19
Ptolemy	28/8/137(3)	1771337	—7;0	—7;5,51
al-Battānī	1/3/888(2)	2045460	4;10	4;32,37
al-Ṣūfī	1/10/964(2)	2073433	5;42	5;40,31

(1) Beginning of 50th year of 3rd 76-year period of the era of Callipos, which began 28/6/—329.

(2) (56, vol. 1, p. 293).

(3) Era Antonini.

(4) Hipparchan longitudes are 2;40 less than Ptolemy's

(5) This is in accordance with al-Ṣūfī's supposition regarding Menelaos' catalogue.

reference point, while al-Battānī's figure suggests that the point is some 0;22 East of the star. Considering however that al-Battānī was a contemporary of Thābit, while al-Ṣūfī lived a century later, and that by Thābit's time doubt had certainly been cast upon the truth of the absolute longitudes of Ptolemy and Hipparchos, although not upon the difference 2;40, we conclude that Thābit meant his zero point to be some 0;22 East of ζ Piscium.

Having established the position of the point P_1 in relation to the stars, it is now natural to translate the whole model into a motion of the equator in relation to the stars. Instead of P_1 rotating about Υ_0, the reverse obtains, and the equator now passes through Υ_0, and the point diametrically opposed, intersecting the ecliptic at the point Υ. The 'fixed ecliptic' of the original model is now a moving great circle passing through Υ_0 and the solstitial point B distant 90°, and the angle $\overline{B\Upsilon_0\Upsilon}$ is maintained constant equal to ε_0. These features are shown in Fig. 3 where

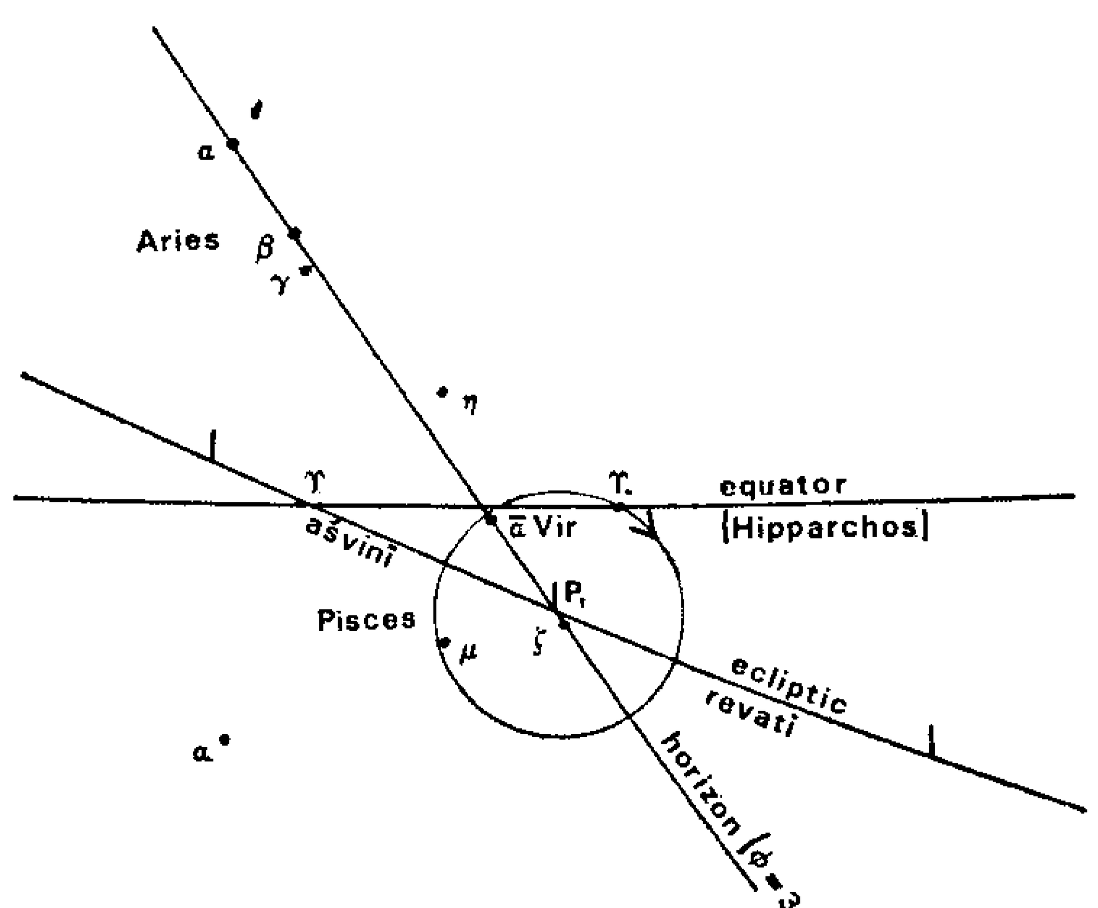

Fig. 3. Star map showing certain stars in Aries and Pisces, together with the ecliptic, the equator at the time of Hipparchos, and the position of the horizon at a latitude of 36° which serves as an alignement of α, β Arietis, α Virginis and ζ Piscium. The stars are placed according to Ptolemy. The small circle is the locus of the head of Aries in the ninth sphere, according to Thābit's model of trepidation.

the small circle is drawn in a star map, showing the stars α, β, γ Arietis, α, η, μ, ζ Piscium and 'anti-Spica', the point ᾱ opposite to Spica, α Virginis, situated according to Ptolomaic coordinates. Since the orientation is that of a star map, not a celestial globe, the point Υ_0 rotates clockwise around P_1. The equator is drawn in the position it would occupy in 130 B.C.

The small circle intersects the ecliptic at the point occupied by the equinoctial point in A.D. 870, and it may be the case, as suggested earlier, that Thābit determined the radius in this way.

It is also clear from the diagram that anti-Spica lies very nearly on the circle. Indeed, according to Ptolemaic coordinates ᾱ has, relative to ζ, the coordinates (3;40, —2;10), so that the distance is

$$\{(3;40)^2 + (2;10)^2\}^{\frac{1}{2}} = 4;15,32 \simeq r = 4;18,43.$$

The point lying on the ecliptic 0;9,48 West of ζ is distant exactly r from ᾱ.

According to the motus formula $\theta = 0$ in the year A.D. 604 and this date is quite different from the date A.D. 562–3 when the Toledan and Khwārizmian tables are free of error. Some light is shed on this discrepancy by considering the date when the equator actually passes through ζ Piscium, or alternatively through the point 0;10 east of ζ and having the same latitude. We recall that the equinoctial point in A.D. 562–3 has a longitude 0;10 greater than that of ζ. According to modern reckoning, which gives the star a latitude —0.239° and zero longitude when $t \doteq 1931167$ (A.D. 576), we find that the equator passes through ζ when the equinoctial point is 0.546° west of the point having the same longitude as ζ, and this occurs when $t = 1945507$ (A.D. 615), and further that the equator passes through that point which lies 0;10 east of ζ and has the same latitude as ζ, when $t = 1941134$ (A.D. 603).

It is difficult to decide whether these complex considerations have much bearing on the reconstruction of Thābit's model. Should one conclude that the centre of the circle lies south of the ecliptic, or perhaps that the observations on which the model is based require such a position, even through astronomers at the time believed it to be on the ecliptic? (According to the Sūrya Siddhānta (viii. 9) the latitude of the junction star of revatī is zero.) An exact recalculation would seem to be relevant only to observations actually carried out at the time. It is important to bear in mind that an observation of the altitude of a star at the time of meridan transit is more accurate than any other, and perhaps unknown to us, such observations were carried out around A.D. 600. Note that the equator at the time of Hipparchos passed very nearly through Spica.

The amplitude of trepidation employed by Thābit, 10;45, may have been intended to represent the interval along the ecliptic to the point culminating with the star β Arietis. In any case the distance from the point 0;20 East of ζ Pisc to the point culminating with β Arie is 10;37, if the calculations are made with Hipparchan coordinates.

The theory of trepidation seems to be appropriate to a system of star mapping framed by the horizon and meridian, in which the principle observations are of rising, setting and culmination. This will be borne out very strongly in the next section, where the historical origin of the sidereal zero point is explained. We note also that observations of the Sun on the meridian play a fundamental role in the practical determination of the movement of the eighth sphere. Indeed the use of the polar longitude depends on a conversion from the observations of the transit of the star to a point in the luni-solar motion which progresses uniformly along the ecliptic. At the moment when the star, having polar longitude L culminates, the Sun's position is known with respect to the sidereal origin, which is information of the kind required in the construction of horoscopes, in which one needs the position of the Sun, Moon and planets with respect to the ascendant.

[To be continued. Fig. 6 and Tables I and II, referred to in this part, will appear in the next issue.]

STUDIES IN THE MEDIEVAL CONCEPTION OF PRECESSION

(Part II)

RAYMOND MERCIER, SOUTHAMPTON

5. *The Greek star catalogue and the sidereal ecliptic*

We may discover through the study of Hipparchos' Commentary on the Phenomena of Aratos, how it was that a point near ζ Pisc came to be selected as the zero point of the longitude in the Indian systems and those descended from them. In this Commentary Hipparchos describes the rising of the constellations, as seen by an observer at a latitude of 36°, giving in particular the points of the ecliptic rising simultaneously with the beginning and end of the constellation, and also the stars culminating at those times. Within the format of the Commentary he does not give those stars which set simultaneously, but it may be observed that Spica sets with the rising of the beginning of Aries; Aratos only noted that Ara sets as Aries rises. Hipparchos marks this rising with the emergence of η Pisc ('the forefoot of the Ram', 41, p. 255), and in fact α, β, γ Arie and η Pisc all rise nearly at this time, while Spica sets; ζ Pisc also rises, but Hipparchos does not mention this star. He gives the point of the ecliptic which rises simultaneously as 18½ ♓ (i. e., —11;30, i. e., —8;50 with respect to Ptolemy's origin). This point is not far from ζ Pisc, and with a little study one can see that the setting of Spica is exactly simultaneous with the rising of our sidereal zero point at this latitude. Encouraged by this, I calculated the point of the ecliptic rising with each of these stars as a function of latitude, both for the positions given by Ptolemy with the longitude reduced by 2;40, and for those positions calculated for this time by Peters and Knobel for their edition of Ptolemy's catalogue. The results are shown in the graphs in figs. 4 and 5.

The longitude of this point, denoted λ_0, is given by the expression

$$\cot \lambda_0 = \frac{\tan\varphi \sin\varepsilon}{\sin\xi} - \frac{\cos\varepsilon}{\tan\xi}$$

where

$$\xi = \sin^{-1}(\tan\varphi \tan\delta) - \alpha$$

and where α, δ are the right ascension and declination of the star, φ is the geographic latitude and ε is the obliquity. The obliquity is taken as 23;51 for the Ptolomaic positions, and as 23.71 for the calculated positions. These formulae contain in fact, in modern style, the same process of calculation as developed by Ptolemy (Almagest viii. 5), who calculates first the arc ξ, which is the interval between the equinoctial point, and the point where the horizon intersects the equator. The right ascension and declination may be expressed easily in terms of the longitude λ, latitude β and polar longitude L,

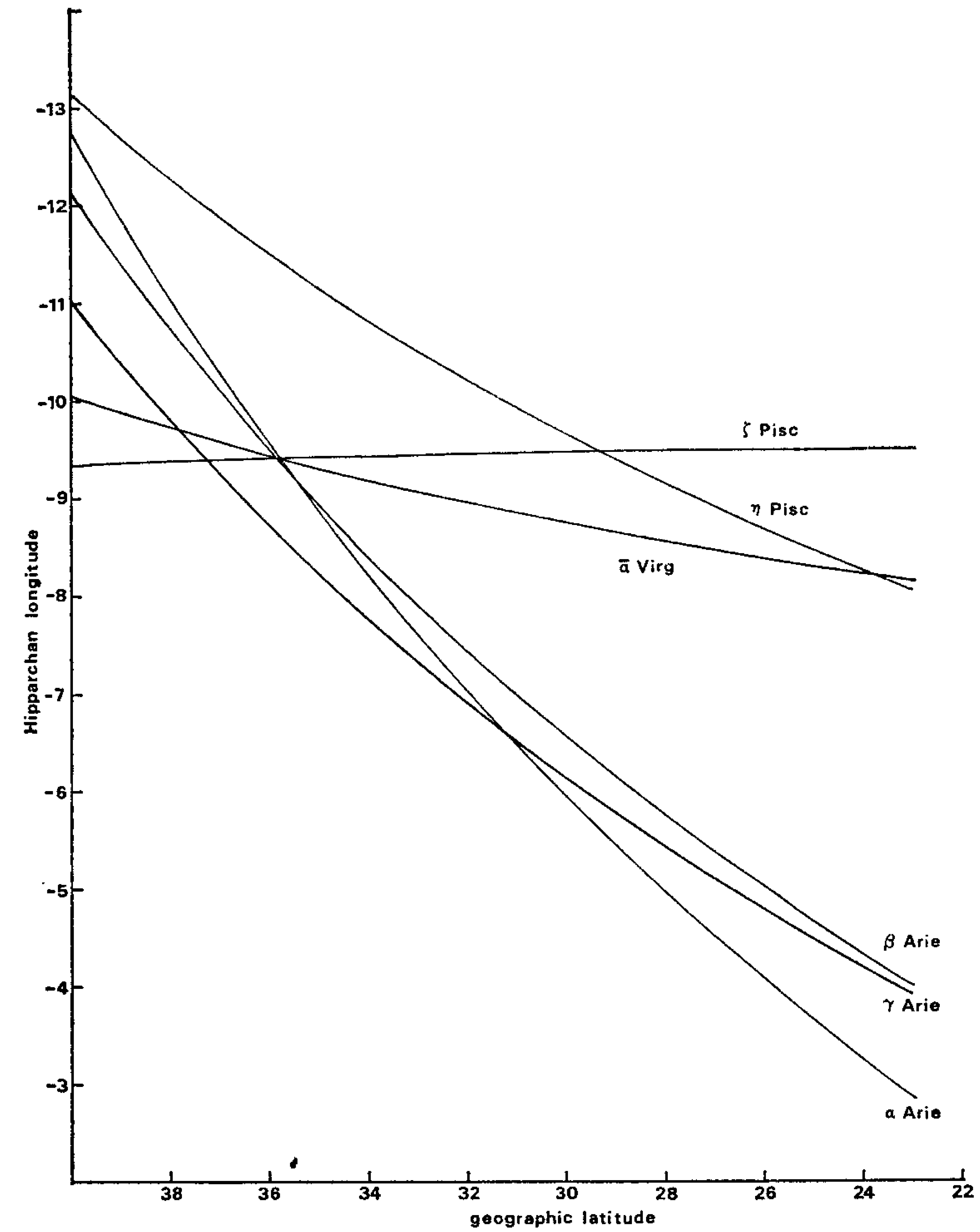

Fig. 4. Graphs of λ_0 versus φ for various stars, showing the point of the ecliptic rising with each of the stars, as a function of geographic latitude. These curves refer to the positions of the stars at the time of Hipparchos, as taken from Ptolemy's Catalogue with the longitude reduced by 2;40.

$$\tan\alpha = \tan L \cos\varepsilon$$
$$\cos\delta = \cos\beta \cos\lambda/\cos\alpha.$$

The polar longitude L is given by

$$\tan L = \tan\lambda - \tan\varepsilon \tan\beta/\cos\lambda$$

and this may be derived from a more general formula which is also needed in the discussion. If we have two stars with ecliptic coordinates λ_1, β_1 and λ_2, β_2 then the great circle through them intersects the ecliptic at a point having longitude λ_I given by

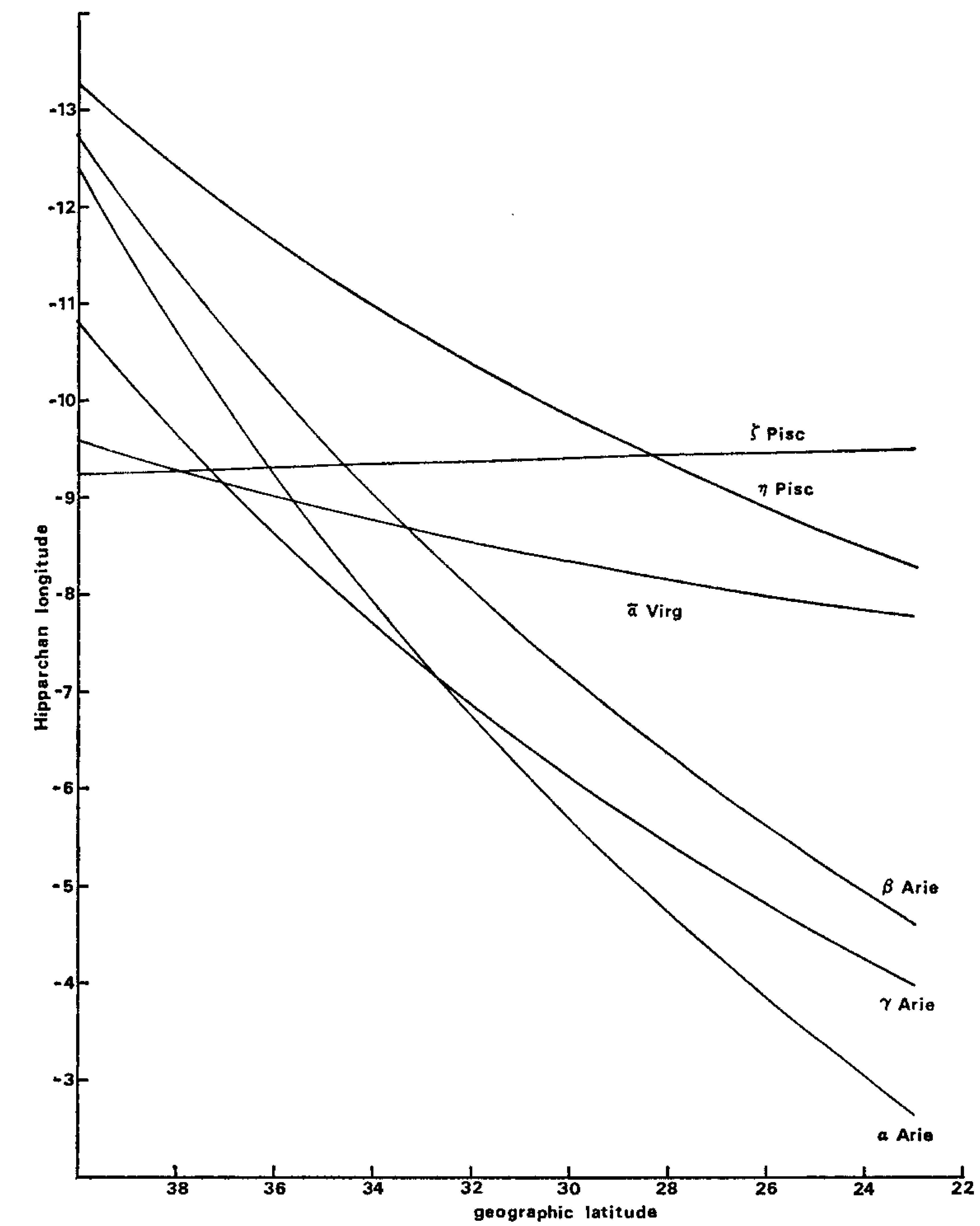

Fig. 5. These curves differ from those in fig. 4 only in referring to positions of the stars in 130 B.C. as calculated by Peters and Knobel.

$$\tan\lambda_I = \frac{\sin\beta_1 \sin\lambda_2 \cos\beta_2 - \sin\lambda_1 \cos\beta_1 \sin\beta_2}{\sin\beta_1 \cos\lambda_2 \cos\beta_2 - \cos\lambda_1 \cos\beta_2 \sin\beta_2}$$

This is most easily derived using, not spherical trigonometry, but vector methods. Let $\mathbf{n}_1$ and $\mathbf{n}_2$ be unit vectors terminating at the stars, and let $\mathbf{n}_e$ be the normal to the equator. The $\mathbf{N} = \mathbf{n}_e \times (\mathbf{n}_1 \times \mathbf{n}_2)$ is the vector in the direction of the intersection sought, and $\tan\lambda_I$ is the ratio between the appropriate components of $\mathbf{N}$.

If we let one of the 'stars' in this last argument be the pole of the equator, so that $\lambda_1 = \pi/2$ and $\beta_1 = \pi/2 - \varepsilon$, we obtain the formula for tan L, which would also be difficult to obtain otherwise.

Returning now to fig. 4 it is seen that not only does Spica set simultaneously with the rising of the origin of the sidereal ecliptic, but we also have then the rising of α,β Arie and ζ Pisc. The striking coincidence of these curves at one point shows that these four stars were assumed to be in exact alignment. We ought to say 'assumed' for it is clear from fig. 5 that the alignement does not hold for the stars in the positions calculated for that time, but is peculiar to the Greek catalogue. In the next Table the longitude λ_I is shown, calculated for the various stars taken in pairs.

pair in alignment	λ_I: Hipparchan coordinates	λ_I: calculated coordinates
α Ar, β Ar	—9;18,43	—14;36,46
α Ar, α Vi	—9;23,39	—8;56,33
β Ar, α Vi	—9;23,55	—8;40,16
γ Ar, α Vi	—9;40,45	—9;8,15
α Ar, ζ Pi	—9;23,3	—9;18,40
β Ar, ζ Pi	—9;23,2	—9;20,16
γ Ar, ζ Pi	—9;22,2	—9;17,17
α Vi, ζ Pi	—9;23,6	—9;16,28

Ptolemy's coordinates are all multiples of 0;10, indicating some rounding-off procedure. This uncertainty entails a 'thickness' for each curve of about 0;10: The intersection of the four curves may then be taken as

$$\lambda = -9;23 \pm 0;5, \qquad \varphi = 35.8 \pm 0.2$$

Ptolemy (Almagest vii.1) describes a number of alignments[1] on a much smaller scale, but gives no indication that he is aware of this one. Indeed it must be one of the best kept secrets of Greek Astronomy. In contrast to the alignments which Ptolemy describes, it would be hard actually to observe this one between diametrically opposed points except with the aid of the horizon as the circle of alignment, at the particular latitude of 36°, since Spica is so far from the close spaced pair in Aries. At an early stage in the development of stellar mapping this observation would have been especially valuable in controlling the results of triangulations over smaller patches of the sky, and their transfer to the globe. Surely such an alignment preceeds in a natural way the use of the principal great circles defined more abstractly.

This alignment would be remarkable enough simply as part of the structural history of the Greek star catalogue, concerning as it does stars of such importance near the equinoctial points. However this great circle of alignment, the Hipparchan horizon at the moment when the head of Aries rises and Spica sets, also marks the rising of the point of the ecliptic which is the zero point of our sidereal ecliptic. We have seen that this point lies between 0;12 and 0;22 East of the

[1] The Greek is σχηματισμός, which also means 'aspect' in astrology, as in the *Tetrabiblos*, i.2.

supposed position of ζ Pisc, essentially the same point as determined by the various considerations concerning trepidation and the Khwārezmian and Toledan Tables.

We have no direct documentation to show how this information was preserved in post-Hipparchan astronomy, other than the precious Indian reference to the junction star of the lunar mansion *revatī* lying 0;10 West of the zero point. This Indian statement suggests that fairly accurate work was carried out in terms of this alignment. Whether the calculation was made by geometry, or with the aid of a well made globe or planisphere, it would have been far more at home in the context of Hellenistic than in Indian astronomy. The coordinates of the junction stars in the Indian works hardly bear comparison, with regard to accuracy, with those of Greek and Arabic catalogues, and it cannot be supposed that star mapping was ever taken seriously in Indian work, in spite of their use of sidereal longitudes. Thus the sidereal longitudes which one has come to regard as 'Indian' are, like so much besides, of Hellenistic origin, and indeed find the fullest practical explanation at only one latitude, that of Hipparchos. The zero point defined in this way by the horizon reminds one of course of the construction of the ascendant in horoscopic astrology, and the zodiacal signs of the sidereal ecliptic correspond to the houses (*loci*, τόποι) measured from the ascendant.

A Babylonian text of the fifth century B.C. (Br. Mus. 86378), which was discussed in great detail by Kugler (48, pp. 1–72) gives heliacal risings, simultaneous risings and settings, and other precious information concerning stellar positions and constellations. At the end of the list of sixteen simultaneous risings and settings we have according to Kugler's reading (48, pp. 22, 32, with important corrections on p. 171),

Piscis Austrinus, especially Formalhaut } rising
Perseus }
Virgo } setting
Lupus }

These are references to constellations, not individual stars, but would seem to entail the recognition of Formalhaut rising while Spica sets. Note that this list gives not heliacal observations, but simply observations of the night sky, such as Hipparchos gives. The date of the heliacal rising of Formalhaut is however given elsewhere in this text (48, p. 33) as XII.15, and according to Kugler's calculation this is very nearly the time of the Spring Equinox in A.D. —500; elsewhere in this text we are told that the heliacal rising of α,β Arie takes place on I.1 (48, pp. 3, 5, 48). This text preceeds Hipparchos by some 300 years, and does not present observations of the degree of accuracy needed in a star catalogue, but it does testify to a practise of observation of risings and settings, etc., and of α Arie and Spica in particular, well established by the time of Hipparchos.

This Babylonian list of heliacal risings stands at the head of an ancient tradiation of calendars in which risings, settings and culminations are listed against the days of the year, sometimes with associated weather records. These include the Greek parapegmata, the calendar in Geminos' Eisagoge, Ptolemy's Phaseis and the Arabic *kitabu'l-'anwa'* (*'anwa'* = pl. of *nau'*, dawn rising or setting). For example in 'Harib's Calendar', or the 'Calendar of Cordova', we have the setting of Spica and the rising of *Baṭn al-Ḥūt* on 6th April (69, p. 66) (*Baṭn al-Ḥūt*, the Belly of the Fish, a conspicuous ring of stars in Andromeda and Pisces consisting of β, μ, ν, π, δ, ε, ζ, η Andr and ψ^1, ψ^2, ψ^3, χ, φ, ν, τ, 82 Pisc, much larger than the northern Fish of Piscium. See al-Ṣūfī, 78, pp. 118, 196)[2].

The star Spica stands in a special relationship to the Indian calendar. The Indian solar year begins when the Sun has zero longitude, measured in the sidereal ecliptic, while their luni-solar year begins with the last new Moon in the solar year, with the month *Caitra* after the lunar mansion *Citra* (= Spica) with which the full Moon is in conjunction (on the average) in the middle of the month. The last month of the solar year is also called *Caitra*.

Finally, note the remarks made in 9.1 on the possible roles of η Pisc and Spica in determining the zero point of the sidereal longitude in the later Babylonian ephemerides.

5.1 *The identification of the stellar coordinates in the Indian sources*

We have seen that the star ζ Pisc occurs in the alignement discovered above, and there can therefore be no doubt that it was deliberately made part of the convention which determined the sidereal longitude. However the first reference to this star in this connection was made by Colebrooke as part of his masterful study of the Indian asterisms or nakṣatras (22, vol. 2, p. 281 seq). The star is listed by Ptolemy as the thirteenth in Pisces, with magnitude four. It is also mentioned, by implication, in Geminos' Eisagoge (35, p. 39), who gives the number of stars in the two 'threads' leading down from the Fishes to their junction point α Pisc, viz., five in the Northern thread and nine in the Southern. These numbers include α Pisc in both cases, and in the Southern the 'detour' through 80, 89 Pisc (Ptolemy's 14,,15) is omitted, so that the Southern one consists of α, ξ, ν, μ, ζ, ε, δ, 51, 41 Pisc. It appears of course in all the Medieval Catalogues derived from Ptolemy, those of al-Battānī, al-Ṣūfī and al-Bīrūnī, as well as Ulūgh Beg's. The latitude is given accurately by Ptolemy as —0;10, but in the Arab catalogues this appears as —6;0, through a simple omission of the prime on the letter ϛ′ representing one sixth. In his preface (78, p. 195) to this part of his catalogue, al-Ṣūfī notes that the latitude can hardly be —6;0, and explains that in fact it has the same declination as ε Pisc, which would make its latitude +0;22. Nevertheless he gives —6;0 in his table, and in this, as in the rest, he is followed by al-Bīrūnī.

It is therefore unlikely that Arab or Latin astronomers, who were so dependent on these versions of Ptolemy, would actually have understood that this star marked the zero point used in the systems derived from India. Indeed al-Bīrūnī

[2] al-Ṣūfī (78, p. 118) remarks that this last mansion is sometimes called *al-risha*, the 'thread' which properly includes the thread of five stars descending through η Pisc, referred to as *rikis* in the Arsacid Babylonian texts (46, pp. 29, 231).

Table I.

θ or φ	E_I	ε_I	E_{II}	ε_{II}	c
—90	—10;39,43	23;58,40	—10;42,42	23;51,39	—10;41,1
—80	—10;30,02	23;58,19	—10;31,51	23;53,58	—10;32,5
—70	—10;01,41	23;56,16	—10;02,09	23;54,53	—10;05,2
—60	—9;15,23	23;52,44	—9;14,35	23;54,53	—9;19,1
—50	—8;12,17	23;48,10	—8;10,35	23;53,24	—8;15,3
—40	—6;54,03	23;43,05	—6;51,56	23;50,48	—6;56,1
—30	—5;22,50	23;38,07	—5;20,51	23;47,24	—5;23,5
—20	—3;41,17	23;33,52	—3;39,51	23;43,35	—3;41,2
—10	—1;53,07	23;30,51	—1;51,51	23;39,49	—1;52,1
0	0	23;29,26	0	23;36,34	0
10	1;52,36	23;29,48	1;52,15	23;34,14	1;51,4
20	3;41,34	23;31,53	3;41,24	23;33,05	3;39,3
30	5;23,24	23;35,27	5;23,52	23;33,17	5;20,1
40	6;54,53	23;40,03	6;56,21	23;34,48	6;51,0
50	8;13,17	23;44,28	8;15,49	23;37,26	8;09,1
60	9;16,22	23;50,04	9;19,47	23;40,53	9;13,0
70	10;02,28	23;54,17	10;06,20	23;44,42	10; 0,0
80	10;30,28	23;57,16	10;34,11	23;48,27	10;30,1
90	10;39,43	23;58,40	10;42,42	23;51,39	10;41,1

Table II.

θ	E	δ	ε
5	0;55,52	0;22,40	23;56,17
10	1;50,36	0;44,31	23;44,17
15	2;45,16	1;06,45	23;49,48
20	3;39,23	1;27,20	23;28,23
25	4;31,12	1;48,04	23;30,18
30	5;22,30	2;09,21	23;40,37
35	6;09,06	2;28,06	23;41,49
40	6;53,12	2;45,55	23;43,30
45	7;36,35	3;02,38	23;38,24
50	8;14,00	3;17,45	23;40,12
55	8;47,48	3;31,40	23;43,34
60	9;17,44	3;44,46	23;51,32
65	9;43,53	3;54,19	23;45,43
70	10;05,30	4;02,48	23;44,56
75	10;22,47	4;09,08	23;41,46
80	10;35,01	4;14,28	23;44,38
85	10;42,13	4;17,30	23;45,40
90	10;45,00	4;18,43	23;46,19

expresses his perplexity on this point when attempting to identify the Indian asterisms (13, vol. 2, pp. 83–5). He remarks also that Indians themselves whom he consulted were of no help.

In the one Arab catalogue, that of Abū'l-Ḥasan, in which this sidereal longitude is employed, there is no mention of ζ Pisc (see § 8).

In the Sūrya Siddhānta the twenty-eight 'junction stars' (yogatārā) of the asterisms are given in terms of the polar longitude and polar latitude. We are told also the number of stars in each asterism, together with certain vague information about the relative position of the junction star in the asterism. From this meagre data Colebrooke made the first attempt to identify precisely each of these stars. His work was reviewed, and modified slightly, by Burgess in his translation of the Sūrya Siddhānta (19, p. 181 seq). Burgess converted the Indian coordinates L, ℓ to ecliptic coordinates λ, β and made his comparison with the ecliptic coordinates of stars according to Flamsteed's *Catalogus Brittanicus*, having subtracted 15;42 from the longitudes of the latter so as to reduce the epoch to A.D. 560, which was supposed to be the approximate date of the Indian 'observations'.

In Table III we give the Indian coordinates, the corresponding values of λ, β,[8] together with the Hipparchan coordinates (Ptolemy's with the longitude reduced by 2;40) of the star supposed to be the correct identification of that in the Indian list. The Indian coordinates marked (1), (2) are quoted by Colebrooke from the Brāhmasphuṭasiddhānta and the Siddhānta Śiromaṇi, respectively. The number following the name of the star is serial number in Ptolemy's list.

The identification given follows Burgess except in the case of no. 1, *aśvinī*. Both Colebrooke and Burgess expressed great uncertainty as to whether α or β Arie were intended, neither star corresponding at all nearly to the Indian position. It is suggested now that while the list generally gives correct polar coordinates derived approximately from Hipparchos, in the case of ásvinī the Hipparchan ecliptic coordinates of α Arie are preserved.

The choice of α Arie is interesting also because the true polar longitude is essentially equal to the sidereal motion of the Moon. In the uniform divisions of the ecliptic corresponding to the nakṣatra then ζ Pisc begins the first, and α the second; see fig. 3.

Believing that the Indian coordinates were genuine observations made in the sixth century, Burgess and Colebrooke did not make any comparison with the Greek catalogue. We have however seen that the sidereal longitude is determined by a convention peculiar to Hellenistic methods, and this is the reason for the inclusion of the Hipparchan coordinates in Table III. This argument is reinforced by two points: 1) the clarification of the coordinates of α Arie; 2) the precession curve determined by the Sūrya Siddhānta passes exactly through the Hipparchan origin (see § 9.4.1).

The coordinates of Spica, no. 14 *citra*, given by the Sūrya Siddhānta are such as to place it in longitude diametrically opposite to the zero point, although the later Indian works place it much nearer to its correct position.

Considering only the entries for the Sūrya Siddhānta, and omitting nos. 1, 14,

[8] The calculation is made most conveniently by making use of Burgess' formulae

$$\lambda = \ell + \sin^{-1} (\tan \beta \cot \psi),\ \beta = \sin^{-1} (\sin \ell \sin \psi)$$

where $\psi = \tan^{-1} (\cot \varepsilon \sec L)$. Burgess' values were in error by a few minutes occasionally, although this has no effect on the process of identification.

Table III.

	Indian				Hipparchos		Modern	
No. naksatra	L	ℓ	λ	β	λ	β		Δλ —9;20
1 aśvinī	8	10	12;3	9;9	8;0	10;0	α Ari ext. 1	—5;17
2 bharaṇī	20	12	24;40	11;4	17;0	11;10	35 Ari ext. 4	—1;40
3 kṛttikā	37;50	5	39;10	4;43	39;30	4;30	19 Tau 30	0;20
1,2	37;28	4;31	38;58	4;16				0;8
4 rohiṇī	49;30	—5	48;7	—4;48	40;0	—5;10	α Tau 14	—1;13
1	49;28	—4;33	48;12	—4;22				—1;8
2	49;28	—4;30	48;13	—4;19				—1;7
5 mṛgaśīrṣa	63	—10	61;1	—9;48	54;20	—13;50	λ Ori 1	—2;39
6 ārdrā	67;20	—9	65;49	—8;52	59;20	—17;0	α Ori 2	—2;51
1,2	67	—11	65;6	—10;50				—3;34
7 punarvasu	93	6	92;52	6;0	84;0	6;15	β Gem 2	—0;28
8 puṣya	106	0	106;0	0;0	98;40	—0;10	δ Cnc 5	—2;0
9 āśleṣā	109	—7	110;0	—6;56	100;40	—13;10	δ Hya 2	0;0
1,2	108	—7	108;57	—6;56				—1;3
10 maghā	129	0	129;0	0;0	119;50	0;10	α Leo 8	—0;10
11 pūrva phalgunī	144	12	139;54	11;17	131;30	13;40	δ Leo 20	—0;56
1,2	147	12	142;46	11;14				1;56
12 uttara phalgunī	155	13	150;6	12;3	141;50	11;50	β Leo 27	—1;4
13 hasta	170	11	174;26	—10;5	164;0	—12;30	δ Crv 5	1;6
14 citrā	180	—2	180;49	—1;50	174;0	—2;0	α Vir 14	—2;31
1	183	2	183;48	—1;50				0;28
2	183	—1;45	183;42	—1;36				0;22
15 svātī	199	37	182;48	33;44	174;20	31;30	α Boo ext. 1	—9;52
16 viśākhā	213	—1;30	213;31	—1;24	201;20	—1;40	ι Lib 5	2;51
1	212;5	—1;23	212;34	—1;18				1;54
2	212;5	—1;20	212;33	—1;15				1;53
17 anurādhā	224	—3	224;55	—2;52	213;0	—1;40	δ Sco 2	2;35
1,2	224;5	—1;44	224;36	—1;39				2;16
18 jyeṣṭhā	229	—4	230;7	—3;50	220;0	—4;0	α Sco 8	0;47
1,2	229;5	—3;30	230;3	—3;22				0;43
19 mūla	241	—9	242;54	—8;48	234;50	—13;20	λ Sco 20	—1;16
1,2	241	—8;30	242;48	—8;19				—1;22
20 purva-aṣāḍhā	254	—5;30	254;40	—5;28	245;0	—6;30	δ Sgr 2	0;20
1,2	254	—5;20	254;39	—5;18				0;19
21 uttara-aṣāḍhā	260	—5	260;23	—4;59	252;40	—3;10	σ Sgr 6	—1;37
22 abhijit	266;40	60	264;7	59;58	254;40	62;0	α Lyr 1	0;7
1,2	265	62	260;51	61;55				—3;9
23 śravaṇa	280	30	282;32	29;54	271;10	29;10	α Aql 3	2;2
1,2	278	30	280;2	29;56				—0;28
24 śraviṣṭha	290	36	296;12	35;32	285;50	32;0	β Del 4	1;2
25 śatabhiṣaj	320	—0;30	319;50	—0;28	312;10	0;10	λ Aqr 24	—1;40
1	320	—0;18	319;54	—0;17				—1;36
2	320	—0;20	319;54	—0;19				—1;36
26 purva-bhadrapada	326	24	334;43	22;27	324;2	19;40	α Peg 4	1;23
27 uttara-bhādrapadā	337	26	347;25	23;57	345;10	26;0	δ Peg 1	—7;5
28 revatī	359;50	0	359;50	0	350;20	—0;10	ζ Psc 13	0;10
1,2	360	0	360	0				0;20

27 (which is subject to some gross error), the average value of $\Delta\lambda - 9;20$ is $-0;6$, with a variance of 1;33. This result is entirely consistent with the view that the zero point coincides with that determined by the alignement found in § 5, the great circle passing through α, β Arie, ζ Pisc and Spica.

Burgess (19, p. 211) found that his longitudes for A.D. 560 were less than those of the Sūrya Siddhānta by a mean interval 0;56, and concluded that the latter were observed around A.D. 490. One must always be wary of any attempt to infer a date of observation from sidereal longitudes, and although we can now be sure, explained in § 3, that important Sassanian observations were made in A.D. 556 which led to the adoption of 564 as a 'canonical' year for the Khwārezmian Tables, in the case of the junction stars we take the view that the coordinates were calculated at the time of Hipparchos.

There is not in fact much coherence between the Indian star longitudes, and the sidereal longitudes found in the various Indian systems, in view of the dates of zero deviation calculated so thoroughly by Billard. Only in the case of the Brāhmasphuṭasiddhānta, and then only as a gross average, are the mean longitudes actually measured from a point near to ζ Pisc[4].

This is not the place at which to enter into a discussion of the extremely rich and difficult subject of the origin and diffusion of the 'lunar mansions'. Historians originally debated the problem of the origin in terms of Indian and Chinese sources, and later introduced the question of an ultimate Babylonian source. In fact the fifth century cuneiform tablet we have mentioned already provided Kugler with evidence for a Babylonian origin of the mansions, for we find there a list of seventeen constellations, headed by the Pleiades and entitled 'stars in which the path of the Moon lies' (48, p. 70). This part of the debate concerns the earliest stage, and naturally Greek astronomy has no bearing on it. However the eventual development leading to the choice of 28 well defined asterisms and their junction stars comes later, perhaps as late in Indian and Chinese literature as the time of Hellenistic astronomy. The realisation therefore that Hellenistic methods are very clearly involved with the Indian junction stars adds a new dimension to the whole argument, and one is no longer free to argue for a simple generation of the mansions in either Indian or Chinese astronomy, or even for a direct transmission from Babylon. Either the junction stars were selected in Hellenistic astronomy and then transmitted to India, or the selection was made in India and assigned coordinates by people schooled in the detailed trigonometrical techniques of the Hipparchan school. Although Weinstock (99) has argued for a Greek knowledge of the mansions[5], the latter alternative seems to fit better. It has long been clear that some knowledge of Hellenistic astronomy reached India, and again

[4] Note also that the star ζ Pisc is not part of the large oval of stars constituting *Revatī*, an oval which must be identical with the Arabic *Baṭn al-Ḥūt*, and deriving, one would suppose, from the Babylonian *Anunitum* (as described by van der Waerden in his study of the tablet Br. Mus. 86378, 94, p. 14, fig. 3).

[5] He argues from a linguistic analysis of a twelfth century Greek text, and from a Greek magical papyrus of the third or fourth century A.D., in which 28 animal symbols are given in association with the Moon.

we point out that the model of precession in the Sūrya Siddhānta exactly agrees with the Hipparchan origin.

6. *The role of Jupiter in trepidation and the Khwarezmian Tables*

The importance of the years A.D. 562/3 in the Khwarezmian Tables cannot be discovered from the internal structure of the solar tables. However the outstanding astronomical event of that time is the conjunction of Jupiter with the Sun at the time of the Spring Equinox of 564, and I had already argued at some length in an earlier version of this paper that this event was crucial in fixing the parameters of the Tables. This conjecture is now fully justified by the passage in the Qānūn'l-Mas'ūdī given in § 3.1, where we saw that observations of Jupiter were carried out under Khusrau Anūshirwān in 556 by way of fixing a cycle of Jupiter for the interval 552–564.

The mean position of the planet is easily found from the Khwārezmian Tables.

JUPITER	Khwārezmian	Toledan
mean rate	0.0830 9791 0722	0.0830 9082 9754
radix at 1948438	330;16,49	331;39,37
period in days	4332.2388	4332.6080
time of zero long. in 564	1927134.45	1927116.03
long. at Equinox in 564 (1927135.80)	—0;6,42	—1;38,33

The times of zero longitude are respectively 1.3 and 19.8 days before the Equinox of 564.

According to Tuckerman's Tables the true longitude vanished at 1927130.75, while the conjunction was at 1927137.45, respectively 5.05 days before and 1.65 days after the Equinox.

Observations of Jupiter pay a major role in ancient and medieval calendars, and seem to have served to link observations of the day and night sky, so important obviously in the phenomenon of precession.

Kugler (47, pp. 166–9) in his studies of Babylonian tablets of Jupiter's motion found that the planet was used as a 'calibration marker' in a sense, regulating the lunar calendar in Babylonian astronomy, and later Pannekoek (68) and van der Waerden (93) discovered in these tablets the Babylonian use of 'tithis', the uniform lunar day so familiar from Indian astronomy.

Heliacal risings of Jupiter were the basis of a twelve year cycle used in India in the fifth and sixth centuries, and inscriptions of that time have been found in which the date was expressed in terms of this cycle, as Diksit (28) and Fleet (34) have shown. Might there not be some historical connection between this calendar, and the contemporary methods used by Khusrau's astronomers, since both involve a twelve year cycle, and conjunctions of Jupiter with the Sun?

Considerations relating to this planet seem also to be relevant to Thābit's model of trepidation. Essentially five parameters are needed to fix the model in relation to empirical data, the sidereal location of the point P_1, the period and epoch of the *motus*, the radius (or alternatively the maximum excursion of the *equatio*), and the obliquity ε_0. We have seen in § 4.5 that the model fits remarkably well at three points at least which were known to him, and if he demanded agreement with Hipparchos, Ptolemy and also some Arabic measurement, his model would be fixed, for we ought to assume that the sidereal reference point was given, and not open to adjustment, while the obliquity similarly is a received quantity. He may also, as we have seen, have wished to make the maximum excursion coincide with the point of the ecliptic culminating with β Arietis.

This ought to be a sufficient explanation therefore of the parameters, and yet it is possible to argue that there is a relation between some aspects of the model and the motion of Jupiter. The twelve year cycle mentioned in § 3 was geared to the synodic period of the planet, but there was another Jupiter calendar also used in India, a sixty year cycle, based on the sidereal period of the planet. In fact successive years of the cycle are marked by the passage of the mean planet through successive signs at sidereal longitudes 0°, 30°, 60°, . . . (24;81, p. 32).

In order to understand better the bearing this might have on the development of models of trepidation we compare the actual 60 year calendar with one which would be obtained if precession were taken into account. The aim is to show what might have been done by people who attempted to take into account precession in terms of its effect on this calendar.

If the planet is referred to the moving equinox, then the dates of passage through the signs are progressively advanced and after some 1000 years, when the precession amounts to 15°, the disparity between the new-year dates of the sidereal and tropical Jupiter calendars is greatest, and afterwards decreases linearly to zero, as the precession increases to 30°. At that point the year numbers of the two Jupiter calendars will differ by one, but the 'local error' due to precession will be zero. This displacement of the new year date, with respect to the new year date of the sidereal Jupiter calendar might have been conceived as a zig-zag function ranging from 0° to 15° and back to 0° on a base of some 2000 years. Various models of trepidation using such functions are well known, and will be mentioned later, and it is possible that all such models arose in this way, or related ways. In other cases one might have studied the progression through lunar mansions instead of signs, and one might have interpreted precession in terms of its effect on conjunctions of Jupiter and Saturn (Great Conjunctions), instead of Jupiter and the Sun. In any case zig-zag functions arise naturally when the accumulated precession is projected out onto signs or lunar mansions.

If we take as a prototype of Thabit's model, a zig-zag function of range 15° on a base of 2028.5 years, the slope would be 53.24″ p.a. This is quite close to the value 54″ per sidereal year quoted in the Sūrya Siddhānta, where one has a zig-zag function of range 54° on a base of 7200 years.

One would therefore look for a relation between the sidereal period of Jupiter and the period of trepidation, and indeed a simple calculation confirms that the period 1481800 days is very nearly 342 sidereal periods and 4057 tropical years. Corresponding to the range of values allowed for the period, as found in § 4.4, we would have the ranges,

4332.284–4332.748: sidereal period of Jupiter
365.206– 365.245: tropical year.

The values for the sidereal period of Jupiter known from Indian and Arabic tables fall within this range, although the hypothesis would fit best a value around 4332.7 somewhat larger than those actually encountered. (The tropical period is around 4330.365 days, which is al-Battānī's value.)

Periods of 71 and 83 years[6] were associated with the motion of Jupiter in Babylonian astronomy (46, pp. 43–4), but there one finds also a large period, mentioned in at least one tablet in the British Museum, Sp. II. 985, described by Kugler (46, pp. 48–50). This period is 344 years, a period related in a simple way to the small periods, for we have $344 = 4 \times 71 + 60$, but related also toThābit's period, for we have,

$$4057 = 12 \times 344 + 71 = 47 \times 71 + 60 \times 12.$$

The period of 344 years corresponds to 29 sidereal revolutions of the planet, and if one takes as a Babylonian year 365.26 days (which is approximately the value entailed by the tables, 45, pp. 72, 91), there is implied a sidereal period

$$\frac{344 \times 365.26}{29} = 4332.74,$$

which is the sort of value entailed by our own hypothesis.

Thus the theory of trepidation was not simply an exercise in empirical curve fitting, but an attempt to *explain* the phenomenon of precession in terms of the movement of Jupiter, the calendar marker of the night sky, in its relation to the Sun, the time keeper of the daily sky.

7. *Trepidation and azimuthal oscillation*

In the course of precession the declination of a star will oscillate between the limits $\beta \pm \varepsilon$, and therefore the azimuth of that point of the horizon at which the star rises will also oscillate. Thus Eisler was led to suggest that the notion of trepidation arose from ancient observations of the changing azimuth of stars on the horizon. As he relates (32), he persuaded Böker to produce, with the aid of a precession globe, some curves of azimuthal oscillation for a number of important stars. These results were published in a study which Eisler reviewed for these *Archives*, a review which included many of Böker's interesting curves.

The notion was taken up by Michel (53) in an interesting paper where he attempted to fix a specific connection with Thābit's trepidation. An observer at geographic latitude φ finds that the star rises at a point of the horizon having azimuthal coordinate A, measured northward from the point due East, where,

$$A = \sin^{-1} (\sin \delta \,/\, \cos \varphi).$$

[6] The period of 71 years is the basis of the Hipparchan and other formulations of Jupiter's mean motion. Ptolemy, for example (Almagest ix. 3), states that the angle between Jupiter and the Sun increases by 65×360 in 71 years less 4.9 days.

If the star is situated on the ecliptic, then $\sin\delta = \sin\varepsilon \sin\lambda$. According to his calculation, if $\varphi = 33;19$ as for Baghdad, and $\varepsilon = 23;33$, then A will be $10;45$ (the extreme value of Thabit's trepidation) when $\lambda = 23;33$. He remarks that this is 'plus qu'une coincidence'. But it is not even that, for one must take $\lambda = 22;58$ in order to make $A = 10;45$. This is of course just inconsequential number fiction, and it would prove nothing even if the arithmetic were correct.

Michel is on more interesting ground however when he draws attention to the 'horizon' mentioned in Thābit's treatise and to the circle thus labelled in his diagram. Certainly the horizon plays no role in the geometric analysis leading to the dependence of E upon the *motus*. On the other hand we have seen why the horizon is important in understanding trepidation, for the horizon at a latitude of 36° determines the alignement of the stars which defines the fixed points in the stellar sphere which are carried around the small circles in the model. This may be the reason for Thābit's reference to it, but one must remember that so far we have had no evidence at all an awareness of this alignement in post-Hipparchan astronomy.

On the other hand, there was another, less common, usage of the term 'horizon', when it meant simply the equinoctial colure. This is mentioned by Isaac Israeli in the Yĕsōd hā 'Ōlām (43, p. iv of the first part of the German translation), who carefully distinguishes it from the more usual sense. This usage also occurs in Pedro Nuñez' treatise on the sphere (65, p. 112), where he gives a diagram which includes a circle at right angles to the equator, labelled 'horizonte'. This sense seems to fit rather better in Thābit's work, which opens with a definition of the coodinate frame,

> Imaginabor speram equatoris diei et tres circulos in ea signatos qui sunt circulus orizontis et circulus linee medii celi transiens per duos polos et secans circulum orizontis in ipsis polis, deinde circulum orientis et occidentis, et est ille qui est in medietate eius quod est inter duos polos et dividit speram in duo media. (83, p. 102)

It is clear that no firm argument has been made out for Eisler's suggestion.

8. *The star catalogue of Abū'l-Ḥasan 'Alī ibn al-Marrākushī*

It might be concluded that only Indian works actually give stellar longitudes measured from ζ Piscium, but there is one Arabic work at least to show that that would be wrong. This is the star list included in a work by the thirteenth century astronomer of Western Islam, Abū'l-Ḥasan 'Alī ibn al-Marrākushī, a work translated by J. J. Sédillot (79; the star list is on pp. 146–9). Sédillot unfortunately provided no identification of the stars, complaining that too much work would be involved in making the comparison with Bayer's catalogue. Nevertheless it is easy enough to identify all the 240 stars by comparison with Ptolemy's list. It is then clear that we have a list in which the longitudes are 6;38 greater than Ptolemy's, in other words they are measured from a point 0;22 East of ζ Piscium. They are also listed strictly in order of increasing longitude, without regard to latitude. The list begins:

ζ Ceti, δ Andr, θ Erid, α Pisc, β Andr, γ Arie,
β Arie, β Cass, ν Ceti, δ Ceti, ...

The star ζ Pisc is not listed. The list is dated A.D. 622 (Hijra)[7].

Abū'l-Ḥasan in the same work gives tables of trepidation whose parameters are very similar to Thābit's and which are meant to be used with the solar and planetary tables he provides (the solar tables use the sidereal year 365.26 days). The *motus* formula is

$$\theta = 3;51 + (5409') \times (\text{Arab years}/1000)$$

$$\theta = \frac{360}{1415108}(t - 1933304)$$

while the table of the *equatio* is given very nearly by $E = 10° \sin\theta$. The date of zero *equatio* is 5th Feb A.D. 581. The curve of $E(t)$ is shown in fig. 6.

From the examples which he gives it is clear that he intends that this *equatio* should be added to the longitudes given in his list of stars. However the *equatio* is 0;40 in A.D. 622, and so there is an inconsistency with the date carried by the list of stars. The rectification would be simplest if the date on the list were ignored, or changed to A.D. 581.

Why should Abū'l-Ḥasan have chosen such an unlikely zero point for the sidereal longitude? We recall that the year A.D. 564 is of special importance in the calibration of those Arabic tables following in the Indian/Sassanian tradition, and we know also that al-Ṣūfī's star catalogue had by Abū'l-Ḥasan's time superseded all others. Now according to al-Ṣūfī's model of precession (9.8) we find that the longitude of ζ Pisc was —0;22 in A.D. 564, and conclude therefore that Abū'l Ḥasan calculated his reference in this way, taking as his zero point the equinoctial point of A.D. 564.

The extreme position of the equinoctial point has the Ptolomaic longitude 3;22, which is very nearly equal to the polar longitude of γ Arie. We have seen that in Thābit's model, the extreme point culminated very nearly with β Arie. In § 9.7 there is a reference to late Medieval lists of lunar mansions in which the coordinate of the first mansion is the Alfonsine correction to Ptolemy's longitude plus 3;24.

9. *Notes on models of trepidation and precession*

1. Babylonian zero point. 2. Ptolemy. 3. Theon of Alexandria. 4. Sūrya Siddhānta. 5. Ārya Siddhānta. 6. Siddhānta Śiromaṇi. 7. al-Battānī. 8. al-Ṣūfī. 9. Alfonsine. 10. Copernicus. 11. Modern formula.

The movements of the eighth sphere according to a number of historically important sources are briefly discussed, and all the results are collected in graphical form in fig. 6. The zero point of the graph is the point lying 0;20 East of the Ptolomaic position of ζ Pisc, which we take as the zero point of Thābit's

[7] There may be a reflection of this list in a list of astrolabe stars credited to Messahalla in his Tractatus Astrolabii, Camb. Univ. MS Ii III.3, fol. 71r (38, p. 162), in so far as this list also begins with ζ Ceti. The longitudes are about 15;24 greater than Ptolemy's and refer the stars to the equinox of about A.D. 1220.

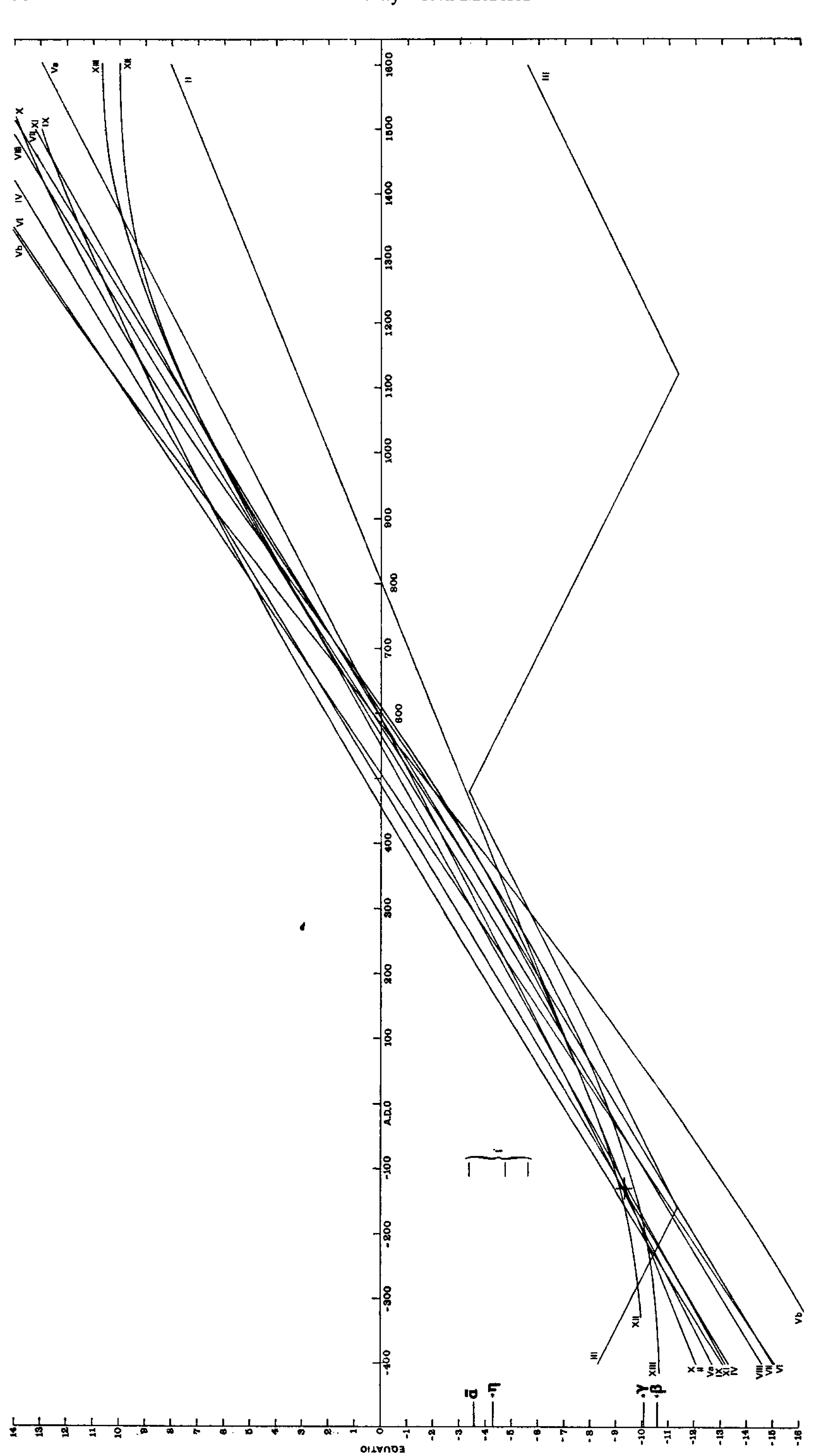

Fig. 6. Curves showing the *equatio* E of precession for the various models discussed in the paper. The reference level is the point of the ecliptic lying 0;20 East of Ptolemy's position of ζ Pisc. Legend: I Babylonian zero of longitude, II Ptolemy, III Theon, IV Sūrya Siddhānta, Va Ārya Siddhānta (linear), Vb Ārya Siddhānta (sine), VI Siddhānta Śiromaṇi, VII al-Battānī, VIII al-Ṣūfī, IX Alfonsine, X Copernicus, XI Modern, XII Abū'l-Ḥasan, XIII Thābit ibn Qurra. The positions of anti-Spica ($\bar{\alpha}$), and the points culminating with β, γ Arietis and η Piscium (*β, *γ, *η) are marked, as well as the Hipparchan origin (+).

model, and in the following this point will be denoted E_0. The equinoctial point according to modern calculations coincided with E_0 in A.D. 545, when its longitude was 18;52 (A.D. 1900), since the calculated position of ζ Pisc is 0;6 West of the Ptolomaic position. The positions of anti-Spica (—3;36), and the positions culminating with β, γ Arie (—10;37, —10;4, Ptolomaic) and η Pisc (—4;20) are also shown.

The graphs representing Thābit's and Abū'l-Ḥasan's models are taken from the formulae already quoted.

9.1 Babylonian zero point

Lunar and planetary tablets of Babylonian origin all record longitudes as measured from some sidereally fixed reference point, and Kugler made many calculations in an attempt to fix this point (45, p. 87; 46, pp. 28, 172–4; 47, pp. 513–21). I have made no recalculation of his results, which were based on P. V. Neugebauer's tables, but it is most unlikely that a repetition, making use of Tuckerman's tables would yield anything essentially new. Kugler calculated the position of the equinoctial point of date in terms of the Babylonian longitude, and reduced his result to the standard date A.D.—100. A calculation, using the modern formula, of the equinoctial point of A.D. —100 gives the position 9;2 East of E_0. Thus we may translate Kugler's positions of the equinoctial point, λ_0 measured on the Babylonian scale, into a position E relative to E_0 where

$$E = 360 - 9;2 - \lambda_0.$$

Kugler's positions fall into three separate groups around $E = -3;22$, $-4;46$, $-5;37$ (47, pp. 513–21). More recently Huber (42) has reviewed the topic in the light of texts more recently brought to notice, but without really fixing the point more precisely. His result may be expressed as $E = -4;34 \pm 0;20$, with a standard deviation of 1;8.

Kugler compared (in effect) the figure —3;22 with the longitude of anti-Spica, with which it nearly agrees, and made other comparisons also with the longitudes of μ, ν Pisc. One possibility hitherto overlooked, it seems, is that the culmination of η Pisc marked the meridian passage of the zero point. If one uses the coordinates of this star calculated by Peters and Knobel, we find that its polar longitude is —5;0 (A.D. —129), which implies that $E = 5;0 - 9;20 = -4;20$, which is well within the range of Huber's result. This star is the first in the list of thirty-three Babylonian 'normal stars'. This list was first established by Epping, and is given by Kugler in various parts of his work, 46, pp. 29, 321; 47, p. 550 seq (where he discusses the identification of the description of η Pisc, given in the list as 'the star reached by the band of the fish', a reference found only in late material of the Arsacid period). This list is also examined by van der Waerden (94) and Huber.

We have already seen that Hipparchos marks the beginning of the rise of Aries by the emergence of this star on the horizon. Finally note that the zero point of the Babylonian ecliptic lies very nearly on the circle of Thābit's model.

9.2. *Hipparchos und Ptolemy*

The phenomenon of precession was discovered by Hipparchos who, as we understand it, determined stellar longitudes in his own time, and discovered the change in the longitudes when compared with those fixed earlier by Timocharis. This discovery raises important questions about Hipparchos' ability to determine the position of the ecliptic and the equinoctial point in the night sky. The exact practical location of the ecliptic can be determined only with the aid of observations of lunar eclipses, as the name ecliptic correctly suggests. It is true that a less direct method of locating it could have been through a consideration of the stars exactly at the time of solstice when the ecliptic passed through the E–W points of the horizon, but this method depends on the determination of midnight and the date of the solstice, and is fraught with error. The equinoctial points could be determined as the intersections of the ecliptic with the equator, as drawn for example on a globe. This much would serve for an initial orientation for the reduction of observations, but we have Ptolemy's example (Almagest vii. 1–3) to show that in making precise determinations of the positions of the stars, they were observed in relation to the Moon, and then lunar theory was invoked in the final determination of the coordinates. Dicks has given an interesting critical survey of the problem of fixing the equinoctial points in earlier Greek science (27).

It is essential to bear in mind therefore the following fundamental assumption, and indeed fundamental weakness, of Greek Astronomy, that the 19 year cycle of lunar motion was perfectly coherent with the tropical tear. For Hipparchos is quoted as saying (Almagest iii.1; 72, p. 145), 'We find in 19 years as many months (235) as those people maintained, but on the other hand the year has an additional term, which is at most[8] 1/300 day less than the quarter, . . .'. Thus Hipparchos and Ptolemy assume that a full Moon, for example, in congruent points of the 19 year cycle, occurs always at the same position in relation to the equinoctial point. This is not in fact true, for the equinoctial point precesses from these positions at the rate of about 0.5° per centűry. It is for this reason that Hipparchos' year is slightly too long, for it is designed to fit the 19 year cycle, this length being

$$\frac{126007\frac{1}{24}}{4267} \times \frac{235}{19} = 365\tfrac{1}{4} - \frac{1}{313.68} \text{ days,}$$

since Hipparchos determined the synodic month from the 345 year eclipse cycle. We can see why Hipparchos says 'at most 1/300'; Censorinus (*de die natali*, ch. 18; 20, p. 38) gave the additional term as 1/304, when describing the Hipparchan year.

It was in this way then that Hipparchos and Ptolemy located the equinoctial point in the night sky. The relation between the lunar theory and the equinoctial point had to be completed by a precise fixing of the date of at least one equinox, presumably done by Hipparchos, and with considerable accuracy. When the stellar longitudes were determined with the aid of the lunar theory, a precession rate of 1°

[8] Manitius, who has here 'at least' (mindestens), should be corrected; the text is . . . ἔλασσον τριακοσιοστῷ ἐπιλαμβάνοντα μάλιστα μέρει . . .

per century would be obtained, for the lunar positions at congruent points of the 19 year cycle do indeed precess at that rate. Delambre (25, vol. 2, p. 254) tried to show from an analysis of Hipparchos' observations that a proper comparison between his and Ptolemy's would have yielded a precession rate of 0;49 p.a. This view was firmly refuted by Vogt (92) who determined from the Commentary on Aratos the coordinates of 122 stars mentioned by Hipparchos, and concluded that the differences in longitude from those of Ptolemy well justified the rate of 0;36 p.a.[9].

In Babylonian astronomy the date of the equinox was fixed according to the rule first clarified by Neugebauer (57), by which the tropical year is 12 months 11;3,10 tithis (1 tithi = 0;02 month). Since we have 235/19 = 12 months 11;3, 9,28, . . . tithis, the Babylonian value is a rounding off of the same proportion used in Greek astronomy. More sophisticated developments in both Babylonian and Greek astronomy concern the inequalities related to the eccentricity of solar and lunar orbits, but do not give any improvement over this fundamental assumption about the mean motion.

These features are preserved in the Jewish calendar, for it also is based on the assumption of an exact 19 year cycle, in which the mean synodic month is 29 days $12^{793/1080}$ hours (43, second part, p. 29), and this is essentially a rounding off of the Hipparchan value, which may be expressed as 29 days $12^{792.979/1080}$ hours. There are actually two versions of the calendar using respectively the years $365\frac{1}{4}$ and 365 days $5\frac{3791}{4104}$ hours, and the latter is exactly 235/19 times the synodic month (43, 2nd part, p.i.). With this latter value, which is virtually the same as that of Hipparchos, and obtained by the same method, the average date of 1 Nisan drifts slowly with respect to the Spring Equinox. During the present cycle, 1959–1977, it occurs some 6.5 days later. Coincidence would have obtained around A.D. 300, about the time when this calendar was consistuted in its present form.

The epoch of Ptolemy's catalogue is the era of Antoninus which we take as 28 August 137 = 1771337. The equation of the eighth sphere is, at that time, —6;40.

9.3 *Theon of Alexandria*

A model of precession, or rather trepidation, attributed to Hellenistic astrologers, is reported by Theon of Alexandria (86, p. 53), and precise accounts are also given by Abū Ma'shar (2, Tract II Diff 8; 31, vol. 2, p. 503) and al-Battānī (56, vol. 1, p. 126–7). Many historians have had occasion to discuss this report (31, vol. 2, p. 625; 12, p. 86–7; 52, pp. 446, 485; 58, p. 7). In this model the equation of the eighth sphere measured from some as yet undetermined sidereal reference point, oscillated between 0° and —8°, reaching —8° 128 years before Augustus,

[9] Ptolemy has been accused of outright plagiarism, and of a dishonest fabrication of evidence of precession in support of his stellar coordinates, for example by Böker (15, p. 46) who made a rather superficial attempt to show that the Greeks used a fixed sidereal ecliptic which was taken over by Ptolemy. One might of course criticize Ptolemy for failing to make a determination of the equinoctial point independent of the lunar theory.

and returning to 0° 640 years later in AD 483, so that in Theon's time it is about —1;31. Abū Ma'shar tells us that the amplitude is 5;12,45 at the beginning of the year 265 Yez. This era began on 16/6/632 = 1952063, so that the beginning of 265 is 1952063 + 264×365 = 2048423. This fixes the date of zero amplitude as

$$2048423 - 5;12,45\times 80\times 365 = 1896218 = 25/7/479.$$

One may make an estimate of the stellar situation of this model, but I know of no clear argument leading to a unique position. There is no reason at all to suppose that the Indian-Arabic origin E_0 was intended to be used, and one would be more likely to find some association with the Babylonian ecliptic. This model of precession might have been used to compare predictions of the longitude of the Sun and the planets made according to astrological methods with those of Ptolemy or Theon, these being the only tables known at the time which gave tropical longitudes. We know in fact of a Greek ephemeris of A.D. 349/50, analysed by Neugebauer (59) and Burckhardt (18) in which the longitudes listed for both inner and outer planets are all nearly 1;40 greater than those calculated from Theon's tables[10]. This difference is equal to the amount given by this model of trepidation for A.D. 350, which shows, in case it were doubted, that there were astrological tables with this model was used. However we cannot find the sidereal reference point exactly unless we know the basis of calculation of the emphemeris.
We can argue indirectly however by finding the error in the tropical tables. For the solar tables of Ptolemy one can determine easily an approximate error formula. The true sun, according to Ptolemy or Theon, at the time of the Spring equinox (according to modern calculation) has the longitude

$$-0.00430°n - 0.923°$$

where n = A.D. year number. Taking A.D. 350 as a representative date, the error is then —2;26, and the error in the astrologer's table would be —2;26 + 1;40 = —0;46, which is the distance from the true equinoctial point of A.D. 350 to the sidereal reference point. The former is located —2;47 with respect to E_0, so the desired point is at —3;33. If we had taken an earlier date, say A.D. 1, we would obtain in the same way —2;30. Both μ Piscium (—3;10) and anti-Spica (—3;20) as well as one of the Babylonian zero points fall in this range. In Fig. 6 the zero level is taken as anti-Spica.

Biot claimed to find μ Piscium as the centre of an oscillation of ± 8°, writing,

> Ce mouvement avait atteint sa limit occidentale, *in consequentia*, 128 ans avant l'ère d'Auguste; de sorte que, 28 ans plus tard, au temps d'Hipparque, le point équinoxial était entré dans sa phase de rétrogradation, comme ce grand astronome l'a observé. En partant de ces données, je trouve, par un calcul fort simple, que le mouvement oscillatoire ainsi défini devait s'opérer autour d'une petite étoile de la constellation des Poissons que nous désignons par la lettre μ . . . (12. p. 86)

[10] The Greek horoscopes analysed by Neugebauer and van Hoesen always measure longitude from some point placed at a distance of the order of 4° West of the equinoctial point of date. Although this material is very interesting, it is difficult to derive clear numerical results; Theon's trepidation is brought into the commentary on horoscope no. 81 (60, p. 27).

Perhaps he calculated the longitude corresponding to a Hipparchan precession rate for the interval 640–28 = 612 years, which would put the reference point at a Ptolomaic longitude 2;40 — 6;7 = —3;27, about the same as Ptolemy's longitude (—3;30) of μ Piscium. This is not very logical, but I do not see any other way in which he might have obtained this result. With his usual boldness, Biot may have come nearer the truth than he deserved. Martin (52, p. 485) accepted Biot's claim.

One obvious defect of this version of trepidation is that it is in flagrant disagreement with Ptolemy's report that precession had progressed 2° from the time of Timocharis (c. 295 B.C.) to Hipparchos.

The practice of setting the equinoctial point at 8° of Aries, which is found in the Babylonian system B, and apparently in much of the horoscopic material gathered by Neugebauer and van Hoesen, is presumably related to this model of trepidation. This position of the equinoctial point is situated therefore near E = —11;30; the only stellar alignement one might associate with it is the line joining β Arie with α Pisc.

9.4 Sūrya Siddhānta

A model of trepidation is mentioned in rather cryptic terms in the Sūrya Siddhānta (iii 9.12;10;19 pp. 99–105), a model first brought to the notice of European historians by Davis (23, p. 266) and Colebrooke (22, vol. 2, p. 329). The amplitude oscillates between the limits ±27°, passing through zero in the year 3600 complete of the Kali yuga. The rate is 54″ per year (the year being the sidereal year as used in the Sūrya Siddhānta), and therefore the period of a complete cycle is 7200 years. The Julian date of the time when the amplitude is zero is[11]

$$588463.12 + 3600\times 365.2587564 = 1903394.6 = 19/3/499$$

The sidereal reference point is 0;10 East of ζ Piscium, according to the text (viii.1 seq.; 10, p. 175) where the positions of the nakṣatras are specified, and this point is +0;10 on the graph. The formula is therefore,

$$E = (t - 1903395)\,\frac{0;0{,}54}{365.2587564} + 0;10$$

It is interesting that there is very close agreement between this curve and the Hipparchan position of the equinox. For we can take Hipparchos's date as 1674484 = 27/6/—128[12], and his position of the equinoctial point on the graph is

[11] To find the Julian date corresponding to a Kali yuga date we need to know the epoch of the Kali yuga and the length of the year. The epoch is midnight 17/18 Feb. A.D. —3101 at Ujjain, and allowing a longitude correction *(kṣepa)* of 2;10,14,30 days by which the epoch is set back when working out the solar calendar (81, § 26; the kṣepa allows for the equation of the orbit of the Sun at the time of the equinox), and this reduces the epoch to 588465.21. According to the Sūrya Siddhānta the length of the year is 1577917828/4320000 = 365.2587564 days.

[12] According to Ptolemy (Almagest vii.2), Hipparchos' longitudes are fixed for the beginning of the 50th year of the 3rd cycle of Callipos, that is 201 years after 28/6/—329 = 1601069, which comes to 1674484.

$-6;40 - 2;40 = -9;20$. On the other hand the value from the above formula is $-9;14,2$.

The extreme position of the equinoctial point, 27° East of E_0 may be related to the heliacal rising of α Arie. The *arcus visionis* may be taken as 14°, and when the calculation is made using Hipparchan coordinates and the Hipparchan latitude of 36°, the Sun is situated 28;25 East of E_0. This result would be only slightly different if one calculated for another epoch, or indeed for another near-by latitude. One may suppose that the intention of this model of trepidation was to relate the heavens to the earlier time when the heliacal rising of this star marked the commencement of the year. Kugler for example argued very strongly for this location of the New Year, in his discussion of the fifth century tablet Br. Mus. 86378 (48, p. 5), a tablet frequently mentioned in this paper already. We recall that the list of *nakṣatras* in this work begins with α Arie.

9.5 *Ārya Siddhānta*

A model of trepidation is mentioned by Bentley (9, pp. 138–141) and Colebrooke (22, vol. 2, p. 332) who refer to a certain work called the Ārya Siddhānta. This was at first confused with the Āryabhaṭīya, the fifth century work by Āryabhaṭa I. The Ārya Siddhānta in question however is later, perhaps of the tenth century (80, p. 8), and is attributed to an otherwise unknown author named for convenience Āryabhaṭa II. It is available in a number of Manuscripts, and from two of these it was printed under an alternative title Mahāsiddhānta, by M. S. Dvivedi (6), together with a commentary in Sanskrit and an introduction in English. Colebrooke's information is drawn from Munīśvara's Marīci (A.D. 1603), while Bentley, who as usual does not explain the nature of his sources, must have had access to one of the MSS.

In the following, references for the various parameters are made to the accounts of Bentley and Colebrooke, as well as to the text itself.

There are 578159 cycles of trepidation per kalpa (i. 11; 22, vol. 2, p. 332; 9, p. 139), with an excursion equal to the maximum declination of 24° (iii. 13; 22, vol. 2, p. 332; 9, p. 141). The epoch is defined in relation to the Sindhind system (71, p. 28) which places the origin of the kalpa, whose length is 4320000000 years, 1972944000 (= 4567×432000) years before the Kali yuga (i. 7; 9, p. 139). The creation, at the end of which the universal conjunction takes place, lasts 3024000 years[13] (i. 7; 9, p. 139), so that the conjunction, and in particular zero trepidation, occurs 1969920000 years before the Kali yuga. By the time the Kali yuga is reached the motion is

$$\frac{1969920000 \times 578159}{4320000000} = 263640.504^{\text{rev}}, \text{ exactly,}$$

[13] In the Sūrya Siddhānta the universal conjunction takes place 17064000 years after the beginning of the kalpa, leaving 195588000 years to the Kali yuga. See Burgess's remarks on this figure (19, p. 19).

and the remaining 0.496^{rev} is accomplished in

$$\frac{0.496 \times 4320000000}{578159} = 3706.1085 \text{ years,}$$

which is the Kali yuga date of zero trepidation. (Note that since we have *accessio* at this date, the cycles must have begun with *recessio*.) According to the calendar of the Sūrya Siddhānta the Julian date is

$$588463.12 + (365.2587564 \times 3706.1085) = 1942152 = 28/4/605$$

which is very near to the corresponding date of Thābit's model. The motus θ which increases by 360° during the course of one cycle, is

$$\theta = \frac{360 \times 578159}{1577917542000}(t - 1942152)$$

since there are 1577917542000 days per kalpa (i. 13; 6, p. 139) according to the Ārya Siddhānta.

Now it is difficult to interpret the passage of the work (iii.13) which explains the relation between the equation of trepidation and the motus. Colebrooke and Bentley take the passage to mean

$$\text{equation} = \frac{24}{90}\theta, \quad -90 < \theta < 90, \qquad \text{(a)}$$

whereas according to Dvivedi,

$$\text{equation} = \sin^{-1}(\sin 24° \sin \theta). \qquad \text{(b)}$$

The passage reads,

> *ayanagrahadoḥ krāntijyācāpā kendravaddhanarṇa syāt.*
> The *bhuja* of the equinoctial point is the arc of the sine of the declination, and is added or subtracted as in the case of the anomaly *(kendra)*.

The *bhuja* of an angle A is that part which determines the sine, the zig-zag function which takes the values A, $\pi - A$, $-\pi + A$. $2\pi - A$ respectively in the four quadranst. The 'arc of the sine of the declination' is a quantity which obviously ranges from 0° to 24° since the maximum declination according to this work is 24°. It is not clear however in what way this is understood to depend on the motus. The usual rule for expressing the declination δ in terms of the longitude λ is

$$\sin\delta = \sin 24° \sin\lambda,$$

and if this formula is to be understood to apply to the motus, then of course Dvivedi's interpretation is justified. Indeed, it is hard to see, if the linear motion is understood, why the passage should follow immediately after a section describing the calculation of sines.

The equation of the equinoctial point is to be added or subtracted like the equation of the anomaly of a planet, that is positive in the range $0 < \theta < 180$, negative if $180 < \theta < 360$.

The sidereal origin of the longitude is the junction star of *revatī*, identified as usual with ζ Piscium (xi.2;9, p. 153). From formula (b) we have therefore

$$E = \sin^{-1}(\sin 24° \sin \theta) + 0;20.$$

which is shown in the graph, and which is calculated with the value $\sin 24° = \frac{1397}{3438}$ given in the text (iii.4).

Now the version (a), while not satisfactorily based on the text may well be of historic interest, particularly if it is used together with longitudes measured from the point 0;10 East of ζ Piscium. In this case one has

$$E_0 = \frac{96 \times 578159}{1577917542000}(t - 1942152) + 0;10$$
$$= 0.00003517501\,(t - 1937414).$$

This is interesting because like the model in the Sūrya Siddhānta, it very nearly agrees with the Hipparchan value —9;20 when t = 1674484, giving in fact $E = -9;14,55$ (for the Sūrya Siddhānta at this date $E = -9;14,1$). It is therefore worth considering the possibility that both these linear models were constructed to agree with the Hipparchan star catalogue, while differing only in the rate used. The rate of change of E in the linear interpretation is 46.25″ p.a., and is closer to the rate entailed by the Babylonian and Hipparchan luni-solar data (46.807″ p.a.) than is any other medieval model.

Version (b), which appears to be the correct reading of the text, agrees well with the other Indian curves, at least from the tenth century, which is supposed to be the date of origin of the work, when it intersects the Sūrya Siddhānta curve. The rate is exceptionally high at earlier times, reaching a maximum of 70.5″ p.a.

The extreme position 24° East of E_0 may refer to the heliacal setting of the Pleiades which occurs when the Sun is situated some 22°–24° East of E_0 (depending on the choice of the *arcus visionis*), and this nearly coincides with the (daily) rising of α Pisc at $E = 23;48$[14].

The heliacal setting of the Pleiades may be the proper role of this star in those older lists of the nakṣatras which begin with it, rather than to suppose that they refer to some remote epoch when the equinoctial point was near this star. On the possible role of this heliacal setting in the Harranian Moon-cult, see Lewy (50), and for a very original discussion of its importance in the 'Lion and Bull' motif, see Hartner (39).

9.6 Siddhānta Śiromaṇi

A model of precession, in the proper sense of continuous rotation, is described in Bhāskara II's treatise Siddhānta Śiromaṇi, in the second book entitled Golādhyāya (on the sphere), translated by Wilkinson and Śāstrī (10). Bhāskara, when describing the model (vi.17;10, p. 157), is actually quoting Muñjāla (A.D. 662:

[14] This happens at the 'Spring Equinox' according to Callipos, as quoted in Geminos' Calendar (35, p. 228). That a daily, not a heliacal rising is here intended by Geminos is shown by his use of the verb ἀνατέλλει, instead of ἐπιτέλλει (35, p. 147). The position 24° East of E_0 is actually the midpoint of the Hipparchan sign of Aries, from which point, according to Hipparchos (41, p. 132), Eudoxos measured his longitudes.

10, p. 141). The rate of movement of precession is 199669 revolutions per kalpa (4320000000 years), which amounts to a rate 0.016639083 = 0;0,59,54,2,31,12 p.a.

The epoch of the motion, when the equinoctial point coincided with the stellar reference point, is the beginning of the kalpa, which is taken according to the Sindhind system (71, p. 28) to be 1972944000 (= 4567 × 432000) years before the Kali yuga (10, p. 108). At the beginning of the Kali yuga therefore, the motion amounted to $91188^{rev}299.628°$, leaving 60.372° of the current revolution, which is completed at the date

$$\frac{60.372}{0.016639083} = 3628.325 \text{ Kali yuga}.$$

To convert this to the Julian date, one has to choose one of the Indian calendars and according to the Sūrya Siddhānta we have

$$588463.12 + (365.2587564 \times 3628.325) = 1913741 = 16/7/527.$$

The length of the year according to the Siddhānta Śiromaṇi itself is 365.2584375 (1577916450000 days per kalpa), which is slightly smaller than in the Sūrya Siddhānta, but the difference would make only 2 days difference in the above date.

Colebrooke (22, vol. 2, p. 332) quotes a commentary on this work, the Marīci by Munīśvara (A.D. 1603), who says that in Bhāskara's own time, at the date 1105 Śaka complete = 4284 Kali yuga, the motion amounted to $91189^{rev}10;54,35,23,55,40,48$. This is quite exactly confirmed as the result of the calculation

$$\frac{199669 \times (1972944000 + 4284)}{4320000000} = 91189^{rev}10;54,35,23,55,40,48,$$

which is important in confirming Bhāskara's use of the Sindhind system of placing the epoch of the motion at the beginning of the kalpa.

Concerning the location of the sidereal reference point we have Colebrooke's information taken from the Marīci that it was assumed to be ζ Piscium itself (22, vol. 2, plate facing p. 284). Bhāskara apparently followed Brahmagupta; certainly the latter in his Khaṇḍakhādyaka takes Revatī to begin at ζ (16, p. 149).

Finally therefore,

$$E = \frac{199669 \times 360}{1577916450000}(t - 1913741) + 0;20$$
$$= 0.000045554275\,(t - 1906424),$$

the date when E vanishes being 4/7/507[15].

[15] The date on which the correction vanishes is of course still a problem for the modern Hindu astrologer. In a popular modern handbook (75, p. 51) we are told that the dates A.D. 361, 394, 397, 498, 559 and others have been proposed. See 'Notes added in proof' at end.

9.7 *al-Battānī*

This model of precession uses the rate of 1° in 66 years, where one year = 365.24056 days (56, vol. 2, p. 124), while the epoch of the star catalogue is 1/3/888 = 2045460 (56, vol. 2, p. 369), and ζ Pisc has the longitude 4;10. Therefore

$$\begin{aligned} E \quad & (t - 2045460) \times 0.00004148366 + 4;30 \\ & = (t - 1936984) \times 0.00004148366, \end{aligned}$$

the date when $E = 0$ being 5/3/591.

In Ptolemy's time this puts the equinoctial point 0;11 East of Ptolemy's position.

Luni-solar intercalation, and the 19-year cycle in particular played no role in Islamic culture, and were indeed proscribed in the Qur'ān (ix.36), so that perhaps Islamic astronomers were predisposed to view in a fresh light the problem of determining the tropical year In any case it was shown first of all in the Baghdad school that this year was shorter than the Greek value.

9.8 *al-Ṣūfī*

This uses the same rate as al-Battānī, although it is not clear exactly what value one should assume for the length of the year for his usage, and one might assume al-Battānī's value. At the date of his catalogue 1/10/964 = 2073432 (56, vol. 1, p. 194), the longitude of ζ Piscium was given as 5;42, so that

$$\begin{aligned} E & = (t - 2073432)\ 0.00004148366 + 6;2 \\ & = (t - 1927993)\ 0.00004148366 \end{aligned}$$

the date when $E = 0$ being 23/7/566.

At Ptolemy's time this puts the equinoctial point 0;10 West of Ptolemy's position.

9.9 *Alfonsine*

The Alfonsine tables include a more complicated model than those described so far in these notes, for it is the sum of linear and sinusoidal terms, although the latter is approximated closely by the sine function and so is simpler in this respect than Thābit's formula. Strictly speaking the tables given are for the longitude of the solar apogee, which is assumed to be fixed in relation to the stars.

The linear term *(motus medius augis solis)* increases at the rate of one revolution in 49000 Julian years, and is given by

$$a_1 = \frac{360}{49000 \times 365.25}(t + 1829349)$$

where the constant has been adjusted so as to give agreement with the values listed in the table for the various eras (4, fol a7r).

The sinusoidal term *(equationum motus accessus et recessus sphaere stellate)* is

$$a_2 = 9° \sin \theta$$

where θ *(medius motus accessus et recessus octave sphaere)* is given by

$$\theta = \frac{360}{7000 \times 365.25}(t - 1727038)$$

the zero point at 16/5/16 (not A.D. 15 as in Dreyer, 30, p. 247) being adjusted so as to reproduce the values given for the various eras (4, fol a7r). It is to be understood that the operative date for any era is noon of the day preceeding, so that the Alfonsine era itself runs from noon 31/5/1252. These formulae for a_1 and θ will reproduce exactly all the numbers given for the eras, but the table of the *equationum* differs slightly from the sine function.

The quantity $a = a_1 + a_2$ is the longitude of the solar apogee. To find the equation of precession for our graph note that the Alfonsine star table is constructed from Ptolemy's with a difference of longitude of 17;8, and so ζ Piscium has a longitude of 10;8. Thus our formula is

$$E = a(t) - a(t_0) + 10;28$$

where $t_0 = 2178502$ is the Alfonsine era. The value of $a(t_0)$ is given in the tables as 88;41,1, although a direct evaluation using an exact sine function gives 88;40,35 and an interpolation of the equationum table gives 88;40,8. It makes no perceptible difference to the graph which one uses.

We see from the graph that there is general disagreement with the Arabic results, but that the curve lies nearer to the Indian curves up to c. A.D. 1000. The agreement with Hipparchos is reasonable, since here $E = -8;54$ compared with the Hipparchan value —9;20.

The intersection with al-Ṣūfī's curve is near A.D. 1260. At the Alfonsine era, al-Ṣūfī's curve reaches 10;23,31, only some four minutes greater than the Alfonsine value 10;28. One would obtain the Alfonsine value at 2180301 = 4/5/1257. We know that al-Ṣūfī's tables were translated in 1256 for King Alfonso; see Wegener's discussion of the correct date of Alfonso's star tables (98, pp. 143–7).

The Alfonsine model of precession was of course widely used in the late Middle Ages, and one finds many star tables in which the original Alfonsine coordinates, which differ from Ptolemy's by 17;8, are adapted by the use of this model to a later date. Just to give an example, in Saxl's survey of astrological manuscripts in Rome and Vienna (77) there are incipits in three manuscripts which supply a number of precession constants for various years:

source	date	correction to Ptolemy	corresponding value of E	value of E from formula
(1)	1438	19;4	12;24	12;22,32
(2)	1424	18;56	12;16	12;14,21
(3)	1424	18;56	12;16	12;14,21
(3)	1434	18;59,10	12;19,10	12;20,12
(3)	1464	19;19	12;39	12;38,7
(3)	1500	19;36,4	12;56,4	12;57,48

The sources are (1), Rome, Bibl. Vat., Cod Vat. Latinus 8174, fol 449, (77, I, p. 99); (2), Vienna, Nationalbibliothek, 5318, fol 2, (77, I, p. 133); (3), Vienna, Nationalbibliothek, 5415, fol 217, (77, I, p. 153).

All these figures are in clear agreement with the Alfonsine formula, and many other examples might be found of this application. Apart from star lists proper however, one encounters in some manuscripts lists of lunar mansions, and in these the longitude of the first mansion al-naṭh (cornu Arietis) may be equal to the correction to the Ptolomaic longitude, as in the lists above, but there are also lists of mansions in which the longitude has a somewhat different meaning.

In a fifteenth century manuscript (Br Mus Royal 12. D. 6, fol 63v) for example, there is a list of coordinates of initial points of the 28 mansions, beginning 19;26♈ 2;17♉, 15;9♉, ... The first coordinate is the Alfonsine correction to Ptolemy for the year A.D. 1480, approximately.

On the other hand in the same manuscript (fol 45v) there is a carefully composed list of 28 uniformly spaced mansions, with the names given 'secundum hebreos, syros, latinos' and beginning with the longitude 22;5♈. At the foot of the table is written 'Anno christi 1400 principium prima mansio lune in 22,5 Arietis fixi secundum tables Alfonsi ...' Now in A.D. 1400 the Alfonsine correction to Ptolemy is 12;0,8 + 6;40 = 18;40,8 which leaves an excess of 3;24,52. This puts the first mansion 22;5 — 12;0,8 = 10;4,52 East of E_0.

The same thing is found in a late fifteenth century manuscript (Trinity College Cam., 0.II.40, fol 118v) where the coordinate of the first mansion is given as 22;59 in A.D. 1490. The Alfonsine correction to Ptolemy for that year is 12;52,13 + 6;40 = 19;32,13 which leaves an excess of 3;26,47 and puts the mansion 22;59 — 12;52,13 = 10;6,47 East of E_0. These last two examples are in agreement as to the sidereal location of the first mansion, and the significance would appear to be that the point in question on the ecliptic has the same longitude as the circle of declination passing through γ Arietis in the time of Ptolemy, for the polar longitude of that star is 3;24 as calculated from Ptolomaic coordinates.

9.10 *Copernicus*

The Copernican model of precession is described by him in the de Revolutionibus (iii 5–12), and in ch. 12 an example is given which displays the complete formula. In common with the Alfonsine model the Copernican motion is a sum of linear and oscillatory terms, but all the parameters differ from the Alfonsine model. The linear and oscillatory terms have periods of 25816 and 1717 Egyptian years (365 days) respectively. Copernicus measures his longitudes from γ Arietis (which is the first lunar mansions) and his formula for its longitude λ is

$$\lambda = 5;32 + \frac{360\,n}{25816} - 1;10 \sin\left(\frac{360\,n}{1717} + 13;30\right)$$

where n is the time measured in units of 365 days from 31/12/0 = 1721423. Our formula for the equation is simple $E = \lambda - 13;20$. If we now replace n by $(t—1721423)/365$, we obtain

$$E = -73.5670 + \frac{360\,t}{9422840} - \frac{7}{6}\sin\left[\frac{360\ (t-1697921.56)}{626705}\right]$$

The model was plainly intended to fit Ptolemy's and Hipparchos's longitudes, for at these times we have λ = 4;0,42 and 6;39,25 respectively, very close to their values 4;0 and 6;40. See Moesgaard's discussion (54) of the observational basis of this formula.

In Thābit's time (29/4/870 = 2038944), λ = 17;59,1 and E = 4;39,1, whereas Thābit's equatio of that date is equal to the radius of the small circle 4;18,43 = 4;39,1 — 0;20,18. This indicates that Copernicus might have understood the sidereal reference point of Thābit's model to lie 0;20 West of our adopted reference point, in other words just the star ζ Piscium itself.

Copernicus of course differed from his predecessors in so far as he explained the phenomenon of precession in terms of a movement of the earth rather than the stellar sphere. Duhem has drawn attention however to the fact that this was not really a point of originality with Copernicus, for already in 1386 Albert of Saxony (Lib. II, Qu. VI, fol E3va; 31, vol. 9, p. 359) had reported that such a view was current, and since his work was printed in 1492, was it not likely that Copernicus came upon it? There seems to be little purpose therefore in Ravetz's attempt to explain Copernicus's originality on this point (76). His contribution lay rather in his clear grasp of the distinctive character of the observational and empirical tradition of astronomy that developed continuously from Babylonian origins, which had always emphasized sidereal rather than tropical or 'terrestrial' periods. This tradition had not been adequately comprehended in the Ptolemaic theoretical framework, and he realised that if the sidereal periods of the planets were really of fundamental significance, then inevitably they must be seen as rotating about the Sun in relation to the stars.

9.11 *Modern precession formula*

The modern formula for the rate of precession (7, p. 530) is

$$p = 50.2564'' + 0.0222''\, T \text{ p.a.}$$

where T[16] is a measure of time expressed in units of 36525 days, measured from A.D. 1900.0 = 2415020. The quantity E plotted in the graph must equal the longitude of the point 20′ East of ζ Piscium, which is 18;26 + 0;20 when $T = 0$. The formula for the rate is integrated to obtain

$$E = 18.77 + \frac{5025.64}{3600}\,T + \frac{1.11}{3600}\,T^2$$

where $T = (t—2415020)/36525$.

[16] While our reference (7) specifies the use of the tropical year here, there is no point in maintaining this distinction. Although the tropical is, properly speaking, the interval between passages of the (true) Sun through the equinoctial point of date, in this reference 'tropical year' means the interval in which the 'geometric mean longitude of the Sun,' as defined by Newcomb's formula, increases by 360°. Whatever the exact meaning, it is hardly a suitable unit in which to measure the passage of time, since it is not constant. In the interval of 2000 years since Hipparchos the accumulated difference would be some 14 days, entailing a precession of merely 0;0,2, which is entirely unobservable.

This formula agrees well with the Hipparchan figure —9;20, for it gives $E = -9;24$ when $t = 1674484$.

Appendix. Analysis of Thābit's model

The use of vector algebra rather than spherical trigonometry allows a more flexible analysis. Let the unit vectors **i k j** be chosen so that the terminus of **k** is $\mathbf{n}_0$, the terminus of **j** is the pole of the equator, and the terminus of **i** is the point on the equator having right ascension 90°. Let the pole of the fixed ecliptic be $\mathbf{n}_0$,

$$\mathbf{n}_0 = -\mathbf{i}\sin\varepsilon_0 + \mathbf{j}\cos\varepsilon_0 \tag{1}$$

where $\varepsilon_0 = 23;33$ is the obliquity of the fixed ecliptic, and let **n** be the pole of the moving ecliptic having direction cosines $\cos\alpha$, $\cos\beta$, $\cos\gamma$,

$$\mathbf{n} = \mathbf{i}\cos\alpha + \mathbf{j}\cos\beta + \mathbf{k}\cos\gamma \tag{2}$$

The small circle of radius $r = 4;18,43$ is the terminus P_1 of the unit vector ρ

$$\rho = \mathbf{i}\sin r\cos\theta + \mathbf{j}\sin r\sin\theta + \mathbf{k}\cos r \tag{3}$$

where θ, the *motus*, is shown in Fig. 1.

Let B be the intersection of the fixed and moving ecliptics, at the terminus of ρ_1. With either constraint on the point B we have

$$\mathbf{n}_0\cdot\rho_1 = \mathbf{n}\cdot\rho = \mathbf{n}\cdot\rho_1 = 0 \tag{4}$$

while the preferred constraint (I) is expressed by

$$\rho\cdot\rho_1 = 0 \tag{5.I}$$

and the alternative (II) by

$$\mathbf{k}\cdot\rho_1 = 0. \tag{5.II}$$

Now since $\mathbf{n}_0\cdot\rho_1 = 0$ we can assume the following form for ρ_1,

$$\rho_1 = \mathbf{i}\cos\varepsilon_0\cos\varphi + \mathbf{j}\sin\varepsilon_0\cos\varphi + \mathbf{k}\sin\varphi \tag{6}$$

where φ is some angle related to the *motus* θ.

The conditions $\mathbf{n}\cdot\rho = \mathbf{n}\cdot\rho_1 = 0$ are equations which may be solved for $\cos\alpha$, $\cos\beta$,

$$\cos\alpha = \frac{\cos r\sin\varepsilon_0 - \tan\varphi\sin r\sin\theta}{\sin r\sin(\theta-\varepsilon_0)}\cos\gamma \tag{7}$$

$$\cos\beta = -\frac{\cos r\cos\varepsilon_0 - \tan\varphi\sin r\cos\theta}{\sin r\sin(\theta-\varepsilon_0)}\cos\gamma \tag{8}$$

This completely determines the three direction cosines because we have the additional relation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. In this way $\cos\gamma$ is found,

$$\cos\gamma = -\frac{\sin r\sin(\theta-\varepsilon_0)}{\sqrt{[1-\sin^2 r\cos^2(\theta-\varepsilon_0)+\tan^2\varphi\sin^2 r-2\tan\varphi\sin r\cos r\cos^2(\theta-\varepsilon_0)]}}$$

the sign being chosen so that **n** is directed upward. 9

When the first constraint is used we have from (5.I)

$$\sin r\cos\theta\cos\varepsilon_0\cos\varphi + \sin r\sin\theta\sin\varepsilon_0\cos\varphi + \sin\varphi\cos r = 0$$
$$\tan\varphi = -\tan r\cos(\theta-\varepsilon_0), \tag{10}$$

and when substituted in (7) — (9) we obtain

$$\cos\alpha = -\frac{\cos r\sin\varepsilon_0 + \tan r\sin r\cos(\theta-\varepsilon_0)\sin\theta}{\sqrt{[1+\tan^2 r\cos^2(\theta-\varepsilon_0)]}}$$

$$\cos\beta = \frac{\cos r\cos\varepsilon_0 + \tan r\sin r\cos(\theta-\varepsilon_0)\cos\theta}{\sqrt{[1+\tan^2 r\cos^2(\theta-\varepsilon_0)]}} \tag{11.I}$$

$$\cos\gamma = -\frac{\sin r\sin(\theta-\varepsilon_0)}{\sqrt{[1+\tan^2 r\cos^2(\theta-\varepsilon_0)]}}$$

When the second constraint is used, (5.II), we have

$$0 = \mathbf{k}\,.\,\rho_1 = \sin\varphi,$$

in other words $\varphi = 0$. Therefore,

$$\cos\alpha = -\frac{\cos r\sin\varepsilon_0}{\sqrt{[(1-\sin^2 r\cos^2(\theta-\varepsilon_0)]}}$$

$$\cos\beta = \frac{\cos r\cos\varepsilon_0}{\sqrt{[(1-\sin^2 r\cos^2(\theta-\varepsilon_0)]}} \tag{11.II}$$

$$\cos\gamma = -\frac{\sin r\sin(\theta-\varepsilon_0)}{\sqrt{[(1-\sin^2 r\cos^2(\theta-\varepsilon_0)]}}$$

The equinoctial point ♈ is at the terminus of the vector

$$(\mathbf{j}\times\mathbf{n})\,/\,|\mathbf{j}\times\mathbf{n}| = \frac{\mathbf{i}\cos\gamma - \mathbf{k}\cos\alpha}{\sqrt{[\cos^2\alpha+\cos^2\gamma]}} \tag{12}$$

so that the angle E between ♈ and P_1, the *equatio*, is determined by

$$\sin E = \frac{|\rho\times(\mathbf{j}\times\mathbf{n})|}{|\rho|\;|\mathbf{j}\times\mathbf{n}|} = \frac{|\mathbf{j}\,\rho\cdot\mathbf{n} - \mathbf{n}\,\rho\cdot\mathbf{j}|}{|\mathbf{j}\times\mathbf{n}|}$$
$$= \frac{|\rho\cdot\mathbf{j}|}{|\mathbf{j}\times\mathbf{n}|} = \frac{\sqrt{[\cos^2\alpha+\cos^2\gamma]}}{\sin r\sin\theta} \tag{13}$$

If we now substitute from (11.I) and (11.II) respectively we obtain equations (1a) and (2a) given previously (part I, p. 212).

The obliquity of the moving ecliptic, denoted by ε, is determined by

$$\cos\varepsilon = \mathbf{j}\cdot\mathbf{n} = \cos\beta$$
$$\varepsilon = \beta \tag{14}$$

This yields directly equations (1b) and (2b) given previously.

The angle between Υ and Υ_0, which Carmody (83, p. 95) denoted by c, is given by his formula if one uses the first constraint rather than the second. Therefore in spite of numerous obvious divergences between his interpretation and ours, he has at least read the text correctly on this point. We have

$$\cos c = \frac{\mathbf{k} \cdot (\mathbf{j} \times \mathbf{n})}{|\mathbf{j} \times \mathbf{n}|} = \frac{\cos \alpha}{|\mathbf{j} \times \mathbf{n}|}$$

$$\sin c = \frac{|\mathbf{k} \times (\mathbf{j} \times \mathbf{n})|}{|\mathbf{j} \times \mathbf{n}|} = \frac{|\mathbf{k} \cdot \mathbf{n}|}{|\mathbf{j} \times \mathbf{n}|} = \frac{\cos \gamma}{|\mathbf{j} \times \mathbf{n}|}.$$

Substituting from (11.I) will give

$$\cot c = \frac{\cos \alpha}{\cos \gamma} = \frac{\cos r \sin \varepsilon_0 + \tan r \sin r \cos (\theta - \varepsilon_0) \sin \theta}{\sin r \sin (\theta - \varepsilon_0)} \qquad 15.\mathrm{I}$$

In our notation Carmody's formula is

$$\cot c = \frac{\cot (\theta - \varepsilon_0) \sin \theta}{\sin r \cos r} - \cot r \cos \theta \qquad 16$$

and one may show the identity of (15.I) and (16).

With the second constraint one obtains,

$$\cot c = \frac{\cos \alpha}{\cos \gamma} = \frac{\cot r \sin \varepsilon_0}{\sin (\theta - \varepsilon_0)}. \qquad 15.\mathrm{II}$$

In order to discuss the locus of the pole of the moving ecliptic, the terminus of **n**, we find the components of **n** in a reference frame obtained by rotating from **i j k** through an angle ε_0 about **k**. The components of **n** are then

$$\begin{aligned} \xi &= \cos \varepsilon_0 \cos \alpha + \sin \varepsilon_0 \cos \beta \\ \eta &= -\sin \varepsilon_0 \cos \alpha + \cos \varepsilon_0 \cos \beta \\ \zeta &= \cos \gamma \end{aligned} \qquad 17$$

The locus of **n** may be followed in terms of ξ and ζ, which are measured from the pole of the fixed ecliptic towards the Summer Solstice and Spring Equinox respectively, while η is essentially constant.

For the first constraint,

$$\xi = \frac{\tan r \sin r \cos (\theta - \varepsilon_0) \sin (\theta - \varepsilon_0)}{\sqrt{[1 + \tan^2 r \cos^2 (\theta - \varepsilon_0)]}} \simeq -\frac{1}{2} r^2 \sin 2 (\theta - \varepsilon_0)$$

$$\eta = \frac{\cos r + \tan r \sin r \cos^2 (\theta - \varepsilon_0)}{\sqrt{[1 + \tan^2 r \cos^2 (\theta - \varepsilon_0)]}} \simeq 1 \qquad 18.\mathrm{I}$$

$$\zeta = -\frac{\sin r \sin (\theta - \varepsilon_0)}{\sqrt{[1 + \tan^2 r \cos^2 (\theta - \varepsilon_0)]}} \simeq -r \sin (\theta - \varepsilon_0)$$

while for the second,

$$\xi = 0$$

$$\eta = \frac{\cos r}{\sqrt{[1 - \sin^2 r \cos^2 (\theta - \varepsilon_0)]}} \simeq 1 \qquad 18.\mathrm{II}$$

$$\zeta = -\frac{\sin r \sin (\theta - \varepsilon_0)}{\sqrt{[1 - \sin^2 r \cos^2 (\theta - \varepsilon_0)]}} \simeq -r \sin (\theta - \varepsilon_0)$$

The locus according to (18.I) is a figure of eight, and according to (18.II) it is a straight arc.

The locus of the moving solsticial point, located 90° from P_1, is at the terminus of

$$\begin{aligned} \mathbf{n} \times \rho = \mathbf{i}\, &[\cos \beta \cos r - \cos \gamma \sin r \sin \theta] + \mathbf{j}\, [\cos \gamma \sin r \cos \theta - \cos \alpha \cos r] \\ &+ \mathbf{k}\, [\cos \alpha \sin r \sin \theta - \cos \beta \sin r \cos \theta] \\ &\equiv \mathbf{i} \cos A + \mathbf{j} \cos B + \mathbf{k} \cos C, \end{aligned}$$

and the components of this in the same rotated frame as before are

$$\begin{aligned} \lambda &= \cos \varepsilon_0 \cos A + \sin \varepsilon_0 \cos B \\ \mu &= -\sin \varepsilon_0 \cos A + \cos \varepsilon_0 \cos B \\ \nu &= \cos C \end{aligned}$$

The locus is expressed in terms of ν and μ which are measured from *caput cancri* of the fixed ecliptic towards Υ_0 and the pole of the fixed ecliptic, respectively, while λ is nearly constant. Under constraint I we find,

$$\lambda = \frac{1}{\sqrt{[1 + \tan^2 r \cos^2 (\theta - \varepsilon_0)]}} \simeq 1$$

$$\mu = 0 \qquad 19.\mathrm{I}$$

$$\nu = -\frac{\tan r \cos (\theta - \varepsilon_0)}{\sqrt{[1 + \tan^2 r \cos^2 (\theta - \varepsilon_0)]}} \simeq -r \cos (\theta - \varepsilon_0)$$

so that the locus is a straight arc.

Under constraint II,

$$\lambda = \frac{1 - \sin^2 r \cos^2 (\theta - \varepsilon_0)}{\sqrt{[1 - \sin^2 r \cos^2 (\theta - \varepsilon_0)]}} \simeq 1$$

$$\mu = -\frac{\sin^2 r \cos (\theta - \varepsilon_0) \sin (\theta - \varepsilon_0)}{\sqrt{[1 - \sin^2 r \cos^2 (\theta - \varepsilon_0)]}} \simeq -\frac{1}{2} r^2 \sin 2 (\theta - \varepsilon_0) \qquad 19.\mathrm{II}$$

$$\nu = -\frac{\sin r \cos r \cos (\theta - \varepsilon_0)}{\sqrt{[1 - \sin^2 r \cos^2 (\theta - \varepsilon_0)]}} \simeq -r \cos (\theta - \varepsilon_0)$$

so that the locus is a figure of eight, extended East-West. The figure of eight locus described by (18.I) and (19.II) is very narrow, the ratio of length to width being $2 / r \simeq 26.6$.

LIST OF STAR COORDINATES

Hipparchan coordinates

	λ	β	L	ℓ	α	δ
a Arie	8;0	10;0	3;32,15	10;56,28	3;14,10	12;22,15
β Arie	5;0	8;20	1;17,17	9;7,2	1;10,41	9;38,17
γ Arie	4;0	7;20	0;44,19	8;1,20	0;40,32	8;19,15
η Pisc	—2;20	5;20	—4;4127	5;49 47	—4;17,30	3;56,5
ζ Pisc	—9;40	—0;10	—9;35,38	—0;10,55	—8;47,17	—4;2,45
α Virg	174;0	—2;0	173;7,18	—2;11,3	173;42,15	0;35,29
ᾱ Virg	—6;0	2;0	—6;52,42	2;11,3	—6;17,45	—0;35,29

Coordinates calculated for 130 B.C.

	λ	β	L	ℓ	α	δ
α Arie	8;2	9;54	3;38,47	10;49,11	3;20,21	12;17,6
β Arie	4;24	8;24	0;40,55	9;10,50	0;37,28	9;27,18
γ Arie	3;36	7;5	0;28,18	7;44,24	0;25,55	7;55,47
η Pisc	—2;42	5;15	—5;0,14	5;43,55	—4;35,1	3;43,19
ζ Pisc	—9;46	—0;16	—9;39,4	—0;17,26	—8;51,0	—4;9,22
α Virg	174;16	—1;55	173;25,48	—2;5,28	173;58,49	0;32,45
ᾱ Virg	—5;44	1;55	—6;34,12	2;5,28	—6;1,11	—0;32,45

SUMMARY

These studies are concerned largely with that tradition of empirical astronomy which makes use of the 'fixed' zodiac (fixed in relation to the stars), in contrast to the Hipparchan ecliptic measured from the equinox of date. This tradition, with its associated techniques, extends with a remarkable degree of continuity from early Babylonian observations of risings and culminations, through Hipparchan techniques of star mapping, Indian astronomical systems, Thābit ibn Qurra's model of trepidation, the Khwārezmian and Toledan Tables in Arabic and Latin, down to Copernicus' *de Revolutionibus*.

When a fixed zodiac is used two points need to be settled: (i) the determination of the point of zero longitude, (ii) the nature of the particular model of precession or trepidation used to calculate the position of the equinox of date. It is found to begin with that in both the Toledan and Khwārezmian Tables the true Sun is predicted correctly for the year A.D. 564, at a time when the equinoctial point is found a little to the East of ζ Pisc. Since the model of trepidation elaborated by Thābit ibn Qurra was used with the Toledan Tables, it is concluded that the oscillation of the equinoctial point in that model was centred on the same point, near ζ Pisc. It is well known that this point marked the beginning of the fixed zodiac used in the Indian systems, such as the Sūrya Siddhānta and others.

By making an analysis of the positions given in Ptolemy's star catalogue it has been found that there is a perfect alignement of the stars α Arie, β Arie, ζ Pisc and α Virg; this circle of alignement is the horizon for the latitude of Hipparchos' observatory on Rhodes (36°), and thus it was assumed that at that latitude one would have simultaneously the rising of α Arie, β Arie, ζ Pisc and the setting of α Virg. Moreover the point of the ecliptic rising then is some 0;16 East of ζ Pisc, and so identical with the zero point of the ecliptic as used in the Indian and Arabic Tables mentioned above.

Some additional notes are provided to cover various models of precession and trepidation in detail, including various Indian and Arabic models, and a graph is given in which is shown the movement of the equinoctial point according to each model.

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Notes added in proof

1. When the present work was completed in the Autumn of 1975, I learned from Prof. O. Gingerich that he also had attempted to determine the sidereal situation of Thābit's model. He made use of the assumption that the star lists accompanying some MSS of the Toledan Tables were intended for the year A.D. 1070. The result agreed broadly with mine. ('The accuracy of the Toledan Tables', Owen Gingerich and Barbara Welther, preprint series no. 140, October 1975, Centre for Astrophysics, Cambridge, Mass., and ΠΡΙΣΜΑΤΑ, Festschrift für Willy Hartner, Ed. Y. Maeyama & W. G. Saltzer, Steiner Verlag Wiesbaden, 1977, 151–164.)

2. The authors of the modern Indian popular calendars, the *pañcāṅga*, almost all now use anti-Spica as the origin of the sidereal longitudes. The year of zero precession is in most cases near A.D. 290, but those *pañcāṅga*-makers who follow 'older' methods, use dates near A.D. 499 or A.D. 522. There is no sign that the dates quoted in § 9.6 n. 15 have actually been used by *pañcāṅga*-makers in recent years. A most useful survey is provided in the *Report of the Calendar Reform Committee*, published by the Government of India, Council of Scientific and Industrial Research, New Delhi (1955). The national calendar determined by the Reform Committee adopted, in effect, anti-Spica, and defined the precession of this origin to be 23;15 on 21/3/1956.